STRATEGIC INVENTORIES IN SUPPLY CHAIN CONTRACTS UNDER VARIOUS CONFIGURATIONS OF COMPETITION AND COOPERATION

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This thesis is dedicated to

my parents.

DECLARATION

I hereby declare that the thesis is my original work and it has been written by me in its entirety. I have duly acknowledged all the sources of information which have been used in the thesis.

This thesis has also not been submitted for any degree in any university previously.

Gu Weijia
August 2014

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Summary

Strategic inventories, as opposed to inventories carried for well-documented reasons such as cycle inventories, pipeline inventories, safety inventories, etc., refer to the inventories held purely out of strategic considerations. In this thesis, we first concern ourselves with the roles of strategic inventories under supply chain contracting models when bargaining framework is fully or partially implemented, and study their impacts on trading terms, supply chain performance and coordination. We next address the problem when horizontal competition between supply chains is introduced, and further explore the respective scenarios accordingly.

In the first part of this thesis, we investigate the existence and the effect of strategic inventories for a single supply chain where the supplier and the retailer bargain for the trading terms. For a two-period problem, we consider both the case of bargaining taking place in both periods and the scenario where the two parties bargain only in one period. We compare our results with those for the scenario where the supplier and the retailer trade under a Stackelberg game framework. For scenarios when competition exists in vertical controls, strategic inventories can be used to break suppliers monopoly power and reduce the channel profit loss due to double marginalization effect. Retailer can also be incentivized to hold inventories to in effect enhance her bargaining power when negotiation is to take place. However, if cooperation occurs throughout the entire time horizon, inventories are not held in optimal contract due to a drain of additional holding from the channel profit. On the other hand, when the chain is in a transition phase, supplier intends to avoid such a threat, and the vertical competition is actually intensified.

We then introduce horizontal competition between supply chains into the system and study how the impact of strategic inventories changes correspondingly. Taking into account interactions between two parallel chains, inventories continue to play strategic roles in vertical controls, and other influences are speculated too. Proven to be strategic substitutes to each other, strategic inventories carried by competitive chains partially constitute their respective sales quantities, and the strategic complementarity between sales quantities are thus partially replaced. Consequently, larger sales quantities are realized, the gap to first-best optimal is bridged, and horizontal competition is softened with both chains mutually benefitted. Lastly, inventories are used as a commitment tool of one chain to the other to avoid concurrence of large sales quantities when two-time intra-chain bargaining framework is adopted. Under a decision of holding inventories beforehand, one chain is to substantially commit to a pre-determined sales quantity, in order to sustain the collusive behavior to induce the system to approach the first-best outcome.

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Chapter 1

Introduction

Strategic inventories, as opposed to inventories carried for well-cited reasons such as cycle, pipeline, safety inventories, etc. (cf. [3, 15, 22]), refer to the inventories held by the downstream firm (for instance, retailer) purely out of strategic considerations in a single vertical supply chain positioned in a dynamic model; see [1]. In their model, all the foreseeable conventional reasons to carry inventories are eliminated. Empowered to carry forward inventories across periods, retailer is shown to indeed store inventories in the optimal solution, which, compared to a static model, alters (most likely escalates) both entities' and channel profits, as well as the total consumer welfare.

The study of strategic inventories is related to many models of supplier-buyer interactions included in the supply contract literature. The readers may refer to [6, 19, 11] for excellent literature reviews in this field. The study of non-cooperative play has been emerging recently because the incentives of the supply chain parties are typically not aligned, leading to individually optimal decisions that harm the overall supply chain performance. Early research mainly focused on the static models. For example, Corbett and de Groote [8], Ha [12], and Corbett et al [9] considered that suppliers are not privy to the cost structure of the buyer and optimal contracts for the supplier tend to be quantity discount contracts, and Cachon and Zhang [7] studied a queueing model with information asymmetry on costs. However, dynamic procurement is more commonly observed in practice so inventory dynamics are more essential to supply chain coordination. For the case of infinite horizon, there is a growing body of literature addressing the inefficiencies due to the profit-relevant non-contractible actions of parters; see, e.g., Debo and Sun [10], Taylor and Plambeck [17, 18], Ren et al [16], Tunca and Zenios [20], or Belavina and Girotra [4]. It has been shown that when the discount rate for future profits is sufficiently high, short-term gains from unilateral deviations prevent supply chain collaboration so that long-term collaborative relationships are not sustainable. For the case of finite horizon, Anand et al [1] is one of the first papers that studies strategic inventories for vertical controls in a two-echelon supply with a multi-period setting. The authors showed that buyers optimal strategy is to hold inventories to reduce the supplier's monopoly power and lower future prices, and the supplier is unable to prevent this strategy. Keskinocak et al [14] extended the model from Anand et al [1] to study strategic inventories in a situation where the suppliers first period capacity is limited. Other recent research includes Zhang et al [21], Anand et al [2] and Zhang et al [21], to name only a few. In addition, very recently, Hartwig et al [13] presented the experimental test of the effect of strategic inventories on supply chain performance.

The observation and its auxiliary analysis to the role of strategic inventories in optimal contracting stated in the dynamic model appeal to us primarily due to its resemblance to a bargaining framework of our recall. As postulated by authors, retailer is believed to use her storage of inventories to force supplier to lower the period 2 wholesale price, which seems in nature like a reconstruction of the leader-follower structure and a rise of negotiation. Meanwhile, the differences are rather significant too, a major one being that, a bargaining framework for single chain usually mimics upshots from a centralized system, in which double marginalization ceases. It arouses our suspicion in both the presence and the role of strategic inventories if a bargaining framework, which appears considerably powerful and efficient, is established, will strategic inventories still be held had any form of the cooperation been implemented? Will the change of model structure revise or reverse the role of strategic inventories? These are the typical questions to our concerns.

To answer the above questions, we extend the work of Anand et al [1] on the dynamic leader-follower model by introducing cooperations into the vertical control in the format of bilateral bargaining, to replace or partially substitute the leader-follower structure in the sequential-move game. In this thesis, we first investigate the existence and the effect of strategic inventories for a single supply chain where supplier and retailer bargain for the trading terms. For a two-period problem, we consider both the cases of bargaining taking place in both periods and that the two parties bargain only for once at the start of period 1. Later, we also include the scenarios when supply chain is in transition from a cooperative game to a non-cooperative game and the other way round, and compare our results with the dynamic model.

The next issue we would address is that, although shown to play a powerful role in supply chain coordination for the single chain scenario, when horizontal competition exists — which is usually what to expect in the market — bargaining seems to lose its dominant power. In fact, later in this thesis, we recap on an interesting result that, even the leader-follower setting in which horizontal and vertical competitions both exist could surpass bargaining in terms of the channel profit especially when horizontal competition is intense. Therefore, we would like to further explore how bargaining will affect, and be affected by strategic inventories under a setting of two parallel supply chains, and how will the supply chain performance and coordination change accordingly. We thence carry on the set of studies to a system of two supply chains with horizontal competition incorporated and further inspect how the impact of strategic inventories extends and changes.

For the rest of this thesis, we first present the single-chain models and results, as well as highlight some of our findings and analysis in Chapter 2. Two cooperative models, one in Section 2.1.1 with a one-time bargaining, and the other with bilateral bargaining in both periods as discussed in Section 2.1.2, along with another two transitive models in Sections 2.1.3 and 2.1.4, the former with bargaining in the first period and leader-follower in the second and the other way round for the latter, are delivered together with their optimal contracts. In Chapter 3, five double-chain models incorporated with horizontal competition, as extensions to the dynamic model as well as the above four single-chain models, are to our major interests. Traditional reasons to carry inventories are also absent under a similar set of assumptions, yet we show that inventories are still stored in some optimal contracts, and will emphasize imitations and updates on their strategic roles in comparison with the single-chain models.

Chapter 2

Models and Analysis of Single Supply Chain with Vertical Competition and Cooperation

We first summarize the results of several existing models of a single supply chain to facilitate comparisons to our studies later in this chapter. We consider a supply chain consisting of a single supplier S and a single retailer R for the wholesale and retail of a single product. Throughout the thesis, we normalize the unit production cost to be zero, assume zero lead time and deterministic demand with linear demand curve, and the market clearing price corresponding to a sales quantity q is $p(q) = a - bq$ with a, b fixed over the entire time horizon and known to both business entities. For each unit of inventory, a holding cost $h > 0$ is incurred per period, and salvage value is taken to be zero to eliminate arbitrage. The above assumptions are made for a purpose of excluding the traditional reasons for storage of inventories, yet, in one of the following models, inventories are still chosen to be held strategically in the optimal contract.

To start with, there are a few single-period models, one being the centralized system, namely supplier and retailer are coordinated so that the channel profit is maximized. The optimal sales quantity, known as the first-best optimal, $q^{fb} = a/2b$ and the channel profit $\Pi_C^{fb} = a^2/4b$. A static bargaining framework under which both supplier and retailer negotiate over the wholesale price w and sales quantity q as trading terms, modelled through a generalized Nash bargaining [5], will generate the first-best sales quantity and achieve the first-best channel profit, the allocation of which is governed by the ratio of supplier's and retailer's bargaining powers and realized via the choice of w. More specifically, the Nash bargaining model takes the form:

$$\max_{(w,q)\geq 0} \left\{(\Pi_S - D_S)^{1-\alpha}(\Pi_R - D_R)^{\alpha} \mid \Pi_S \geq D_S, \Pi_R \geq D_R\right\},$$

where $\alpha \in [0, 1]$ denotes the beginning power of the retailer, (D_S, D_R) denotes the disagreement point, Π_S and Π_R represent the supplier and retailer profits taking the form $\Pi_S := wq$, $\Pi_R = (p - w)q$, and Π_C represents the channel profit, i.e., $\Pi_C := \Pi_S + \Pi_R$. Note that the optimal solution $q^* = q^{fb}$ and the resulting channel profit $\Pi_C^* = \Pi_C^{fb}$ is universal for any α and any disagreement point (D_S, D_R).

Another classic static (single-period) model in which supplier quotes a linear wholesale price w followed by retailer responding with a procurement quantity q and retailing at the market clearing price, is naturally a leader-follower game with supplier and retailer taking the roles of up- and downstream firms. The optimal contract, determined sequentially by supplier and retailer to maximize their individual profits, is set as follows: $w = a/2, q = a/4b$ and the respective supplier, retailer and channel profits are $\Pi_S = a^2/8b, \Pi_R = a^2/16b, \Pi_C = 3a^2/16b$. Note that to compare the leader-follower outcome with that to the centralized system, a loss of a quarter of the first-best channel profit arises from the well-documented double marginalization effect: The chain is pushed towards a less coordinated direction when the vertical competition is intensified between up- and downstream firms, leading to a lower sales quantity and thus, a loss for the channel.

To discuss the two-period models, we follow closely the work from [1] with the emphasis on two stylized leader-follower games. On top of extending the time horizon to two period, they introduce dynamics by allowing carrying forward inventories from period 1 to period 2, but specify that all purchase/held on-hand quantities must be sold at the end of period 2. Both retailer's ability to hold, as well as the exact amount of inventories are public information. All the previous assumptions made for single-period models still apply to preclude the traditional types of inventories. Notation-wise, superscript $t = 1, 2$ is used wherever applicable to signify the respective period. For a commitment model, supplier quotes w^1, w^2 both at the start of period 1 and credibly commits to such a price menu over the entire time horizon. Retailer, in period 1 procures Q^1 from supplier at a wholesale price of w^1, sells to the market $q^1 \leq Q^1$ and holds any excess $I = Q^1 - q^1$ as inventories at a unit cost of h. In period 2, retailer purchases Q^2 at a wholesale price of w^2, and sells together with the inventories I to the market of a total amount of $q^2 = Q^2 + I$. The optimal outcome states a zero inventory $I = 0$, and the model degenerates to a duplicate of repeated static game. A more interesting model is that supplier quotes wholesale prices dynamically at the start of respective periods while the rest of the events remain in order. The optimal outcome for such a dynamic model is different from that of the commitment case for a broad spectrum of parameters and suggests a different set of mechanics between the up- and downstream firms in respects. In particular, when the dynamic optimal is diversified from the commitment, $I > 0$ is chosen, Π_S is always higher, and Π_R, Π_C increase as well for a reasonably wide range of parameter values, namely retailer indeed chooses to carry inventories across periods and under most circumstances both entities as well as the channel concurrently benefit from such a strategic move.

Note that the inventories arise purely from incentive concerns, and the authors identify the observable marked-down w_2 as a product of these inventories, i.e. retailer exploits inventories to force supplier to lower the period 2 wholesale price. The chain can usually benefit from the strategic move for double marginalization effect is expected to be diminished oftentimes.

The observation and its auxiliary analysis to the role of strategic inventories in optimal contracting stated in [1] appeal to us primarily due to its resemblance to a bargaining framework of our recall. Meanwhile, the differences are rather significant too, a major one being that, a bargaining framework usually mimics upshots from a centralized system, in which double marginalization ceases. It arouses our suspicion in both the presence and the role of strategic inventories if a bargaining framework, which appears considerably powerful and efficient, is stylized. Will strategic inventories still be held had any form of the cooperation been implemented? Will the change of model structure revise or reverse the role of strategic inventories? These are the typical questions to our concerns.

To answer the above questions, we extend the precedents' work on the dynamic leader-follower model by introducing cooperations into the vertical control in the format of bilateral bargaining, to replace or partially substitute the leader-follower setting in the sequential-move game. We will present our work on models and results followed by the comparisons and analysis in the succeeding subsections.

2.1 Models and Results

For notational simplicity, throughout this chapter of single supply chain, we use $p^t := p(q^t) = a - bq^t$ to denote the clearing price in period t, $t = 1, 2$, if no confusion arises.

2.1.1 Cooperation with One-time Bargaining

Supplier and retailer bilaterally bargain over the wholesale prices w^t and sales quantities q^t for both periods $t = 1, 2$ as well as I, the amount of inventories carried over between periods, all in one shot at the beginning of period 1, in order to maximize their joint utility established in a generalized Nash bargaining game with retailer's bargaining power vis-a-vis supplier indexed by $\alpha \in [0, 1]$. Storage for each unit of inventories is charged h per period. A failure in negotiation leads to a zero-profit for both entities. Let Π_S^t and Π_R^t, respectively, denote the profit function of supplier and retail in period t, $t = 1, 2$, so that $\Pi_S = \Pi_S^1 + \Pi_S^2$ and $\Pi_R = \Pi_R^1 + \Pi_R^2$. The 2-period game is then modelled as follows.

$$\max_{(w^1,w^2,q^1,I)\geq 0,\, q^2\geq I} \left\{ (\Pi_S - D_S)^{1-\alpha}(\Pi_R - D_R)^{\alpha} \mid \Pi_S \geq D_S, \Pi_R \geq D_R \right\},$$

where (D_S, D_R) is the disagreement point. It is natural that the supplier and retailer profits are zeros if they never reach an agreement. Thus, we choose $D_S = D_R = 0$. More specifically,

$$\Pi_S^1 = w^1(q^1 + I), \quad \Pi_R^1 = p^1 q^1 - w^1(q^1 + I) - hI, \tag{2.1}$$

$$\Pi_S^2 = w^2(q^2 - I), \quad \Pi_R^2 = p^2 q^2 - w^2(q^2 - I). \tag{2.2}$$

Recall that $p^t = a - bq^t, t = 1, 2$. This maximization problems has infinite optimal solutions satisfying

$$w^{1*} + w^{2*} = (1-\alpha)a, \quad q^{1*} = q^{2*} = \frac{a}{2b}\,(= q^{fb}), \quad I^* = 0.$$

However, the profits under over all optimal contracts are unique, that is,

$$\Pi_C^{t*} = \frac{a^2}{4b}\,(= \Pi_C^{fb}), \quad \Pi_S^{t*} = (1-\alpha)\Pi_C^t, \quad \Pi_R^{t*} = \alpha\Pi_C^t, \quad t = 1, 2.$$

See the detailed derivation in Appendix A.1.

By implementing a one-time bargaining, the strategic inventory is gone while the first-best optimal is achieved, which aligns with our expectation that a centralized system is effectively realized. Furthermore, retailer's ability in carrying inventories does not virtually change the chain coordination, which indicates such a bargaining is adequately effectual. Nevertheless, we could not help but wonder if the efficacy stems from the bargaining structure itself or, on the contrary, the static nature of the model that parallels the commitment contracting? Such a doubt leads us onto the investigation of next model.

2.1.2 Cooperation with Two-time Bargaining

Unlike the one-time bargaining setting, the double-bargaining model permits two entities to carry out negotiations one at the start of each period. Wherefore, rather than being "static" in a sense as the single-bargaining, the dynamics could now exist and any price gap between periods is possible. We are interested in seeing what the optimal contract would look like and model the negotiations as follows.

Period 2: Maximize the joint utility of profit in period 2.

$$\max_{w^2 \geq 0,\, q^2 \geq I} \left\{ \left(\Pi_S^2 - D_S^2\right)^{1-\alpha} \left(\Pi_R^2 - D_R^2\right)^{\alpha} \mid \Pi_S^2 \geq D_S^2, \Pi_R^2 \geq D_R^2 \right\},$$

where the profits Π_S^2, Π_R^2 take the forms in (2.2) and $D_S^2 = 0$, $D_R^2 = p(I)I$. Here, the disagreement point (D_S^2, D_R^2) are defined this way on the grounds that, when negotiation fails, supplier walks away with nothing while retailer can still profit from the sales of strategic inventories. Note that the period-2 model depends on the inventory quantity I brought from period 1. We hereby use $\Pi_S^{2*}(I)$ and $\Pi_R^{2*}(I)$ to denote the profits of supplier and retailer under the optimal contract in period 2, respectively. This notation will also be used in other two-period models of single chain in the sequel.

Period 1: Maximize the joint utility of profit over two periods.

$$\max_{(w^1,q^1,I)\geq 0} \left\{(\Pi_S - D_S)^{1-\alpha}(\Pi_R - D_R)^{\alpha} \mid \Pi_R \geq D_R,\, \Pi_S \geq D_S\right\},$$

where $\Pi_S := \Pi_S^1 + \Pi_S^{2*}(I)$, $\Pi_R := \Pi_R^1 + \Pi_R^{2*}(I)$ with Π_S^1 and Π_R^1 taking the forms in (2.1). We set $D_S = D_R = 0$ by regulating that a failed negotiation at the start of the entire time horizon will cease the operation of the chain. Alternatively, an assumption of $D_S = D_R = a^2/4b$ is also sensible and will not change the optimal outcome. The optimal contract yields

$$w^{1*} = w^{2*} = \frac{(1-\alpha)a}{2}, \quad q^{1*} = q^{2*} = \frac{a}{2b}\,(= q^{fb}), \quad I^* = 0,$$

and under this contract, the relevant profits are

$$\Pi_C^{t*} = \frac{a^2}{4b}\,(= \Pi_C^{fb}), \quad \Pi_S^{t*} = (1-\alpha)\Pi_C^{t*}, \quad \Pi_R^{t*} = \alpha\Pi_C^{t*}, \quad t = 1, 2.$$

See the detailed derivation in Appendix A.2.

Up to now, we have seen $I^* = 0$ in optimal contracting for both one- and two-time bargaining models, in which centralized coordinations are achieved. In other words, bargaining framework seems way too compelling that it completely retrieves any loss due to double marginalization effect, henceforth, covers the strategic role of inventories and even dominates it. In contrast, under a dynamic leader-follower framework, strategic inventories, although implicitly seen and postulated by [1] as a contracting tool of the downstream firm to acquire a lower future wholesale price quoted by the upstream, has in fact reduced double marginalization and improved channel coordination; for a sufficiently broad spectrum of parameters, strategic inventories appear in optimal contracts. On this account, we intend to continue to inspect the optimal contracts when bargaining is integrated partially to the dynamic model. Furthermore, we care to explore into more details how the inventories play a strategic role in each period respectively, inspired by a perceptive trade-off in retailer's period-1 and -2 profits (for an anticipation of retailer's strategic move of storing inventories, leading to a foreseeable lower period-2 wholesale price, will motivate supplier's raising period-1's wholesale price, causing higher cost for retailer's overall period-1 orders). We could exploit results in Section 2.1.2 that period-by-period negotiations do not trigger storage of strategic inventories and outcross it with a leader-follower setting to rack up two dynamic models to our interests, namely bargaining in the first period and leader-follower in the second demonstrated in Section 2.1.3, and vice versa, as in Section 2.1.4. Veritably, these two models, demonstrating the transition phases from cooperation to leader-follower or the other way round, are also of practical values in operational management. We will first present modelling and results for the former of the two transitive models.

2.1.3 Bargaining + Leader-follower

Now we study a transition from bargaining to leader-follower framework.

Period 2: Presuming an inventory quantity I from period 1 and a wholesale price w^2 quoted by supplier, retailer determines the sales quantity q^2 by maximizing his profit Π_R^1, i.e.,

$$\max_{q^2 \geq I} \left\{ p^2 q^2 - w^2 (q^2 - I) \right\}.$$

Knowing the response curve of retailer denoted by $q^{2*}(w^2)$, supplier determines the wholesale price w^2 by maximizing his profit as

$$\max_{w^2 \geq 0} \left\{ w^2 (q^{2*}(w^2) - I) \right\}.$$

Period 1: Suppler and retailer jointly determine the wholesale price w^1, the sales quantity q^1 and the inventory quantity I, aiming to maximize the utility of profit over two periods defined as follows:

$$\max_{(q^1,I,w^1)\geq 0} \left\{(\Pi_S - D_S)^{1-\alpha}(\Pi_R - D_R)^{\alpha} \mid \Pi_S \geq D_S, \Pi_R \geq D_R\right\},$$

where $\Pi_S := \Pi_S^1 + \Pi_S^{2*}(I)$, $\Pi_R := \Pi_R^1 + \Pi_R^{2*}(I)$ with Π_S^1 and Π_R^1 taking the forms in (2.1), and the disagreement point (D_S, D_R) is defined as the profits that retailer and supplier could achieve when the cooperation fails. Taking into account of the leader and follower's roles, we credit the optimal profits in the static leader-follower game to the disagreement point respectively. To be more specific,

$$D_R = \frac{a^2}{16b} \quad \text{and} \quad D_S = \frac{a^2}{8b}.$$

We end up with the optimal contract as

$$w^{1*} = \frac{(7-5\alpha^2)a^2 - 8(1-\alpha)ah - 16(1+\alpha)h^2}{16(a-2h)}, \qquad w^{2*} = 2h,$$

$$q^{1*} = \frac{a}{2b}\,(= q^{fb}), \qquad q^{2*} = \frac{a-2h}{2b}, \qquad I^* = \frac{a-4h}{2b}.$$

Under this contract, the total profits for channel, supplier and retailer are

$$\Pi_C^* = \frac{a^2 - ah + 2h^2}{2b}, \;\; \Pi_S^* = (1-\alpha)\left(\frac{5a^2}{16b} - \frac{ah}{2b} + \frac{h^2}{b}\right) + \frac{a^2}{8b}, \;\; \Pi_R^* = \alpha\left(\frac{5a^2}{16b} - \frac{ah}{2b} + \frac{h^2}{b}\right) + \frac{a^2}{16b}.$$

See the detailed derivation in Appendix A.3.

2.1.4 Leader-follower + Bargaining

For the transitive model with negotiation in period 2, the existence of the optimal strategic inventory is questioned as its strategic role of forcing supplier to lower the future wholesale price is contingent. Thus, we proceed with the modelling and derivation.

Period 2: Follows exactly the discussion of period 2 under cooperation with Two-time Bargaining in Section 2.1.2 and Appendix A.2.

Period 1: Supplier and retailer will optimize over their respective two-period lump-sum profits sequentially as follows. Given a wholesale price w^1 quoted by supplier, retailer aims to determine the sales quantity q^1 and the inventory quantity I by maximize his total profit over two periods as

$$\max_{(q^1,I)\geq 0}\left\{\Pi_R := \Pi_R^1 + \Pi_R^{2*}(I)\right\},$$

where Π_R^1 takes the form in (2.1) and $\Pi_R^{2*}(I)$ takes the form in Appendix A.2. Knowing the retailer's response denoted by $(q^{1*}(w^1), I^*(w^1))$, supplier aims to determine the wholesale price by maximizing his profit as

$$\max_{w^1\geq 0}\left\{\Pi_S := \Pi_S^1\left(q^{1*}(w^1), I^*(w^1)\right) + \Pi_S^{2*}\left(I^*(w^1)\right)\right\},$$

where Π_S^1 takes the form in (2.1) and $\Pi_R^{2*}(I)$ takes the form in Appendix A.2 with $I = I^*(w^1)$, $q^1 = q^{1*}(w^1)$. Solving these maximization problems results in an optimal contract, taking the form

- If $0 \leq \alpha < 1/2$, then

$$\begin{cases} w^{1*} = \dfrac{2(1-\alpha)}{3-2\alpha}a,\ I^* = \dfrac{(1-2\alpha)a}{2b} - \dfrac{h}{2(1-\alpha)b}\,(>0) & \text{if } h \leq \bar{h}_1, \\ w^{1*} = (1-\alpha)a - h,\ I^* = 0 & \text{if } \bar{h}_1 < h \leq \bar{h}_2, \\ w^{1*} = \dfrac{a}{2},\ I^* = 0 & \text{if } h > \bar{h}_2, \end{cases}$$

  where

$$\bar{h}_1 := \frac{(1-\alpha)(1-2\alpha)a}{3-2\alpha} \quad \text{and} \quad \bar{h}_2 := \frac{(1-2\alpha)a}{2} \quad \left(\bar{h}_1 \leq \bar{h}_2\right).$$

- If $1/2 \leq \alpha \leq 1$, then

$$w^{1*} = \frac{a}{2}, \quad I^* = 0 \quad \forall\, h \geq 0.$$

The detailed derivation can be found in Appendix A.4,

2.2 Comparison and Analysis

We first summarize values of a collection of trading terms and profits for all models relevant to our discussion in Table 2.1 and will highlight a few substantial findings.

Table 2.1: Trading terms and profits for two-period models

	Comm.	Dyn.	Coop.	Bg.+LF	LF+Bg. $(1/2 \leq \alpha \leq 1)$
w^1	$\frac{a}{2}$	$\frac{9a-2h}{17}$	$\frac{(1-\alpha)a}{2}$	$\frac{(7-5\alpha^2)a^2-8(1-\alpha)ah-16(1+\alpha)h^2}{16(a-2h)}$	$\frac{a}{2}$
w^2	$\frac{a}{2}$	$\frac{6a+10h}{17}$	$\frac{(1-\alpha)a}{2}$	$2h$	$\frac{(1-\alpha)a}{2}$
I	0	$\frac{5(a-4h)}{34b}$	0	$\frac{a-4h}{2b}$	0
q^1	$\frac{a}{4b}$	$\frac{4a+h}{17b}$	$\frac{a}{2b}$	$\frac{a}{2b}$	$\frac{a}{4b}$
q^2	$\frac{a}{4b}$	$\frac{11a-10h}{34b}$	$\frac{a}{2b}$	$\frac{a-2h}{2b}$	$\frac{a}{2b}$
p^1	$\frac{3a}{4}$	$\frac{13a-h}{17}$	$\frac{a}{2}$	$\frac{a}{2}$	$\frac{3a}{4}$
p^2	$\frac{3a}{4}$	$\frac{23a+10h}{34}$	$\frac{a}{2}$	$\frac{a+2h}{2}$	$\frac{a}{2}$
Π_S	$\frac{a^2}{4b}$	$\frac{9a^2-4ah+8h^2}{34b}$	$\frac{(1-\alpha)a^2}{2b}$	$(1-\alpha)\left(\frac{5a^2}{16b}-\frac{ah}{2b}+\frac{h^2}{b}\right)+\frac{a^2}{8b}$	$\frac{(3-2\alpha)a^2}{8b}$
Π_R	$\frac{a^2}{8b}$	$\frac{461a^2-254ah+576h^2}{1156b}$	$\frac{\alpha a^2}{2b}$	$\alpha\left(\frac{5a^2}{16b}-\frac{ah}{2b}+\frac{h^2}{b}\right)+\frac{a^2}{16b}$	$\frac{(1+4\alpha)a^2}{16b^2}$
Π_C	$\frac{3a^2}{8b}$	$\frac{155a^2-118ah+304h^2}{1156b}$	$\frac{a^2}{2b}$	$\frac{a^2-ah+2h^2}{2b}$	$\frac{7a^2}{16b}$

	LF+Bg. $(0 \leq \alpha < 1/2)$		
	$h \in \left[0, \frac{(1-\alpha)(1-2\alpha)a}{3-2\alpha}\right]$	$h \in \left(\frac{(1-\alpha)(1-2\alpha)a}{3-2\alpha}, \frac{(1-2\alpha)a}{2}\right]$	$h \in \left(\frac{(1-2\alpha)a}{2}, \infty\right)$
w^1	$\frac{2(1-\alpha)a}{3-2\alpha}$	$(1-\alpha)a-h$	$\frac{a}{2}$
w^2	$\frac{(1-\alpha)a}{3-2\alpha}+\frac{h}{2}$	$\frac{(1-\alpha)a}{2}$	$\frac{(1-\alpha)a}{2}$
I	$\frac{(1-2\alpha)a}{2(3-2\alpha)b}-\frac{h}{2(1-\alpha)b}$	0	0
q^1	$\frac{a}{2(3-2\alpha)b}$	$\frac{\alpha a+h}{2b}$	$\frac{a}{4b}$
q^2	$\frac{a}{2b}$	$\frac{a}{2b}$	$\frac{a}{2b}$
p^1	$\frac{(5-4\alpha)a}{2(3-2\alpha)}$	$\frac{(2-\alpha)a-h}{2}$	$\frac{3a}{4}$
p^2	$\frac{a}{2}$	$\frac{a}{2}$	$\frac{a}{2}$
Π_S	$\frac{(17-37\alpha+24\alpha^2-4\alpha^3)a^2}{4(3-2\alpha)^2b}-\frac{ah}{(3-2\alpha)b}$	$\frac{(1-\alpha)(1+2\alpha)a^2+2(1-2\alpha)ah-2h^2}{4b}$	$\frac{(3-2\alpha)a^2}{8b}$
Π_R	$\frac{(-3+21\alpha-20\alpha^2+4\alpha^3)a^2}{4(3-2\alpha)^2b}+\frac{(1+2\alpha)ah}{2(3-2\alpha)b}+\frac{h^2}{2(1-\alpha)b}$	$\frac{\alpha a^2+(\alpha a+h)^2}{4b}$	$\frac{(1+4\alpha)a^2}{16b^2}$
Π_C	$\frac{(7-8\alpha+2\alpha^2)a^2}{2(3-2\alpha)^2b}-\frac{(1-2\alpha)ah}{2(3-2\alpha)b}+\frac{h^2}{2(1-\alpha)b}$	$\frac{2a^2-((1-\alpha)a-h)^2}{4b}$	$\frac{7a^2}{16b}$

This table shows a collection of trading terms and profits for all five two-period models. In this table, "Comm." means the commitment model, "Dyn." means the dynamic model, "Coop." means the cooperation model with one-time or two-time bargaining, "Bg.+LF" means the "bargaining + leader-follower" model, and "LF + Bg." means the "leader-follower + bargaining" model.

Theorem 2.1. *When full cooperation is conducted between the entities over a horizon of two periods, regardless of whether they bargain one, modelled in Section 2.1.1, or two times in Section 2.1.2, first-best quantities are procured and retailed in both periods while no strategic inventories are stored in either optimal contract.*

From a pure analytical point of view, the optimal strategic inventories vanish because the joint utility function for one-time bargaining, attached by a strictly negative partial derivative in I, is in fact strictly decreasing in I. A consistent observation is made for the two-time bargaining's total channel profit function too. This suggests that under full cooperation, regardless of executing a one-time or two-time bargaining, any storage of inventories would be a bare burnout of the channel surplus essentially due to the storage cost incurred.

The analytical result is in fact in accordance with the managerial insight in the following sense. The bargaining powers simply determine the profit allocation, the total of which completely come from the market sales with the holding cost deducted. In comparison with the standard single-period bargaining model where the first-best solution is established, the storage of strategic inventories would only bring down both entities' profits. Recall that in the dynamic model, strategic inventories play a direct role in forcing supplier to cut down the future wholesale price she is to quote; yet, a gap for wholesale prices across period is anticipated in neither bargaining model, nor is there any intermission in a one-time bargaining where both entities commit to the contract which is pre-negotiated at the beginning of period 1. Taking into account the additional inventory holding cost, any strategic inventory is precluded.

To conclude, by implementing the double-bargaining framework, strategic inventories are no longer in the picture. In fact, either form of the full cooperation achieves the same effect as a centralized system, under which the chain is not incentivized to pay for the extra storage cost while no additional benefit is available. This is rather plausible, now that the retailer is empowered to take part in determining wholesale prices on the entire time horizon via bilateral negotiations. Retailer needs not, and will not store any inventories in exchange for a lower future wholesale price, since bargaining will simply accomplish that. Meanwhile, the channel reaches its first-best profit, especially when no holding cost is drained from the chain.

Moving on to the "bargaining + leader-follower" model, due to the implementation of a bilateral negotiation, we focus on the channel profit for comparisons to other models. Considering the different nature of the two frameworks, we separate investigations on channel profit in periods 1 and 2, attempt comparisons of channel profit in period 2 to other models, and finally end up with some neat analytical results.

Theorem 2.2. *For the transitive model in Section 2.1.3 shifting from cooperation to leader-follower,*

i) first-best optimal is secured in period 1;

ii) strategic inventories indeed trim the channel's loss by alleviating double marginalization; in optimal contract, an amount of $I^* = (a - 4h)/2b$ inventories are carried forward if $a > 4h$ is assumed, and the optimal inventory quantity is chosen to maximize the recovery of channel profit from double marginalization;

iii) even with the inventories holding cost deducted, the optimal channel profit in period 2 still prevails the average-by-period optimal in both the commitment and the dynamic model.

Note that the assumption $a - 4h > 0$ has been made in the dynamic model for the feasibility of strategic inventories.

We list down a collection of profits for comparisons. Superscripts "c", "d", "blf" are used to denote respective quantities in commitment, dynamic and "bargaining + leader-follower" models. The notation $\overline{\Pi}$ represents the average profit over the two periods.

$$
\begin{aligned}
&\Pi_C^{blf,1*} = \frac{a^2}{4b} = \Pi_C^{fb},\\
&\Pi_C^{blf,2*} = \frac{a^2 - 2ah + 4h^2}{4b} \text{ (after deducting the holding cost)},\\
&\overline{\Pi}_C^{c,*} = \Pi_C^{c,1*} = \Pi_C^{c,2*} = \frac{3a^2}{16b},\\
&\overline{\Pi}_C^{d,*} = \frac{\Pi_C^{d*}}{2} = \frac{461a^2 - 254ah + 576h^2}{2312b},\\
&\Pi_C^{blf,2*} - \overline{\Pi}_C^{d,*} = \frac{117a^2 - 902ah + 1736h^2}{2312b} = \frac{(117a - 434h)(a - 4h)}{2312b} > 0,\\
&\Pi_C^{blf,2*} - \overline{\Pi}_C^{c,*} = \frac{(a - 4h)^2}{16b} > 0.
\end{aligned}
$$

The nature of the leader-follower game invokes the presence of strategic inventories, which is shown to contribute to an elevated channel profit; see [1]. In our study of the "bargaining + leader-follower" setting, inventories continue to play this strategic role and further weakens the double marginalization effect.

As a by-product, we also obtain the gaps between respective channel profits and the first-best wherever leader-follower occurs and double marginalization effect applies, and we adopt the notation Π_L with proper superscripts to represent these differences/losses.

$$
\begin{aligned}
&\Pi_L^{blf,2} = \frac{1}{16b}((a-2bI)^+)^2, \\
&\Pi_L^{blf,2*} = \frac{h^2}{b}, \\
&\overline{\Pi}_L^{c,*} = \frac{a^2}{16b}, \\
&\Pi_L^{blf,2} \leq \overline{\Pi}_L^{d,*} < \overline{\Pi}_L^{c,*} \quad \text{with first equality holds when } I = 0.
\end{aligned}
$$

The last inequality shows that by holding a proper amount of inventories, the channel profit in period 2 is strictly better-off than dynamic and commitment cases; in other words, bargaining in period 1 inherently magnifies the strategic role of inventories reducing double marginalization.

In fact, from a managerial point of view, the bargaining framework in period 1 incentivizes supplier and retailer, both being forward-looking, to act so that the channel profit is maximized, i.e. $(q^{1*} = a/2b, I^* = (a-4h)/2b)$ is chosen. The bargaining powers $1-\alpha$ and α determine the allocation of the total channel profit, which is realized via supplier's pricing of w_1.

To discuss into more details, the inventories's role of forcing supplier to quote a lower wholesale price in period 2 is still effective. Hence, anticipating to go into a leader-follower setting, retailer practices her right to preserve inventories and will place the order during negotiation. In period 1, negotiation ensures first-best sales quantity is implemented, and the profit increment in period 2 due to a lessened double marginalization effect sourced from strategic inventories will be allocated to both firms proportional to their bargaining powers. Since no double marginalization occurs in period 1, the overall chain coordination over the entire horizon outperforms that of dynamic leader-follower contracting, yet could not match the centralized chain coordination as in Sections 2.1.1 and 2.1.2.

Theorem 2.3. *For the transitive model in Section 2.1.4 shifting from leader-follower to cooperation, in period 2's bargaining game,*

i) first-best optimal is accomplished;

ii) the profit allocation to supplier and retailer may no longer be proportional to their bargaining powers. On the contrary, when $I \neq 0$ is carried forward, retailer's bargaining power vis-a-vis supplier is effectively enlarged.

From retailer's stand, the initiative to hold strategic inventories (to force supplier to lower next period wholesale price) seemed rather out of the picture due to the fact that a pre-arranged negotiation will occur in the future and the wholesale price will be a mutually agreed decision.

However, an interesting finding shows that allowing strategic inventories changes the negotiation outcome, namely

$$\Pi_S^{lfb,2*}(I) = (1-\alpha)\frac{\left((a-2bI)^+\right)^2}{4b}, \qquad \Pi_R^{lfb,2*}(I) = \alpha\frac{\left((a-2bI)^+\right)^2}{4b} + p(I)I,$$

in comparison with a standard static bargaining result,

$$\Pi_S^{b*} = (1-\alpha)\frac{a^2}{4b}, \qquad \Pi_R^{b*} = \alpha\frac{a^2}{4b},$$

while maintaining the same optimal channel profit under reasonable levels of s-trategic inventories, i.e.

$$\Pi_S^{lfb,2*}(I) + \Pi_R^{lfb,2*}(I) = \Pi_S^{b*} + \Pi_R^{b*} = \frac{a^2}{4b} = \Pi_C^{fb} \qquad \text{if } I \leq \frac{a}{2b}.$$

That being understood, a procurement of strategic inventories in advance will alter the profit allocation in the future period. Such a reverse of right certainly comes at a cost, one being the additional holding cost, which is in fact drained from the channel, the other being the possible and plausible wholesale price gap across periods, plausible in a way that to defend her primitive share of profit in period 2, supplier has an intention to quote a high wholesale price in period 1 to obstruct retailer from storing inventories.

The essence of this potential power play enters as a consequence of an underlying structural reconstruction. Instead of a coordinated system with fixed profit allocation ratio, retailer can now use the inventories to induce a Cournot-like supply-side competition (against the monopolistic supplier) in period 2, which could eventually sanction her a larger share of pie. Alternatively, strategic inventories can be seen as a contracting tool to increase retailer's bargaining power.

Next, we will examine supply chain's performance in period 1.

Theorem 2.4. *For the transitive model in Section 2.1.4, the optimal contract depends on parameters α, the bargaining power index and h, the inventories holding cost.*

i) The model decouples in effect into a static leader-follower and a one-period bargaining with the optimal solutions duplicated, if $1/2 \leq \alpha \leq 1$, or $h \geq \frac{(1-2\alpha)a}{2}$.

ii) When $0 \leq \alpha < 1/2$ and $\frac{(1-\alpha)(1-2\alpha)a}{3-2\alpha} < h < \frac{(1-2\alpha)a}{2}$, the inventory quantity $I^* = 0$ appears in the optimal contract; however, retailer's tendency in holding inventories threatens supplier and causes a mark-up in $w^{1*} = (1-\alpha)a - h$ and a lower q^{1*} consequently. i.e. profit loss due to double marginalization is exacerbated. Profit margin for both entities shrinks, but supplier draws a larger fraction from channel surplus in contrast to static leader-follower game.

iii) An amount of strategic inventories $I^* = \frac{(1-2\alpha)a}{2b} - \frac{h}{2(1-\alpha)b}$ is purchased by retailer in period 1 if $0 \leq \alpha < 1/2$ and $h \leq \frac{(1-\alpha)(1-2\alpha)a}{3-2\alpha}$. The strategic *inventories effectively change both w^{1*} and q^{1*}, and both entities suffer from a further loss of profit.*

In conclusion, in terms of channel profit, supply chain in period 1 underperforms, if not equally well as, the static leader-follower model.

We have seen from the above theorem a reverse of the strategic role of inventories in this particular stylized model Section 2.1.4, namely retailer's ability in carrying inventories intensifies the double marginalization effect and can hurt both entities' profits. In fact, retailer bears a larger fraction of the loss if there is any.

Mathematically, Π_S is a piecewise concave function. To be more specific, the curve consists of two downward-facing parabolae with vertices $\frac{2(1-\alpha)a}{3-2\alpha}$ and $\frac{a}{2}$ connected at the point $(1-\alpha)a-h$. Note that $1/2 \leq \alpha \leq 1$ implies $(1-\alpha)a-h < \frac{2(1-\alpha)a}{3-2\alpha} < \frac{a}{2}$ and thus, Π_S is unimodal with the mode occurred at $w_1^* = \frac{a}{2}$, which suggests the additional procurement revenue from sales of I could not compensate supplier's future loss due to a diminished bargaining power. Meanwhile, note that the optimal $I^*(w^1) = \left(\frac{a}{2b} - \frac{h+w^1}{(1-\alpha)2b}\right)^+$ is decreasing in w^1. With $1/2 \leq \alpha \leq 1$, the optimal wholesale price in period 1 chosen by supplier as if the strategic inventory I vanished (i.e., $w_1 = \frac{a}{2}$) is even higher than that as if the strategic inventories were held (i.e., $w^1 = \frac{2(1-\alpha)}{3-2\alpha}a$). In other words, such a value $w^1 = \frac{a}{2}$ would both optimize the wholesale revenue in period 1 and help to achieve the maximal profit $\Pi_{S_2} = \frac{\alpha a^2}{4b}$ in period 2 by driving I to 0 regardless of h, which leads to an indubitable choice of $w^{1*} = \frac{a}{2}$.

From an economic point of view, when retailer already has a relatively large bargaining power, she has held on to quite a segment of the second period channel profit such that any growth in her portion would not be significant enough for an early procurement to pay off. Hence, retailer carries forward empty inventories regardless, and the dynamic framework decouples in effect into a one-period static leader-follower model and a standard one-period bargaining, the optimal of the former being $w_1 = \frac{a}{2}, q_1 = \frac{a}{4b}$.

When $0 \leq \alpha < 1/2$, firstly recognize that retailer is more motivated than in previous case to hold inventories strategically now that she has a larger fraction of pie to compete for. Anticipating such, supplier is incentivized to take an action to avoid her foreseeable loss, and her final decision is closely related to the holding cost. Under a relatively high holding cost $h > \frac{(1-2\alpha)a}{2}$, the strategic inventories are understood as infeasible and the model again decouples in effect into a static leader-follower and a one-period bargaining.

Recall the optimal inventory quantity $I^*(w^1) = \left(\frac{a}{2b} - \frac{h+w^1}{(1-\alpha)2b}\right)^+$ suggests that supplier could raise w_1 to limit the amount of inventories to prevent/reduce supplier's second period profit loss; however, an increase in w_1 will cut q_1 as well, which also hurts supplier's, retailer's and channel profits. For a relatively low holding cost $h \leq \frac{(1-\alpha)(1-2\alpha)a}{3-2\alpha}$, in order to deprive retailer's storage of inventories, supplier must price rather high in period 1, which hurts her wholesale revenue too much. Hence, supplier will accommodate and simply optimizes her overall profit with the sales of strategic inventories included, and an optimal $w^{1*} = \frac{2(1-\alpha)a}{3-2\alpha}$ is chosen.

As for the case when a moderate holding cost is incurred, i.e. $\frac{(1-\alpha)(1-2\alpha)a}{3-2\alpha} < h \leq \frac{(1-2\alpha)a}{2}$, supplier can and will quote a high wholesale price in period 1 to avoid retailer from holding inventories which is anticipated to be used against supplier herself during period 2 bargaining; but too high a wholesale price is likely to hurt her wholesale revenue too. Such a procurement unit cost $w^{1*} = (1-\alpha)a - h$ is just high enough to refrain retailer from storing strategic inventories, and is thus quoted. Under such a contract,

$$\Pi_S^1 < \frac{a^2}{8b} = \Pi_S^{c,1*}, \quad \Pi_R^1 < \frac{a^2}{16b} = \Pi_R^{c,1*}, \quad \Pi_C^1 < \frac{3a^2}{16b} = \Pi_C^{c,1*}, \quad \frac{\Pi_S^1}{\Pi_R^1} > 2 = \frac{\Pi_S^{c,1*}}{\Pi_R^{c,1*}}.$$

To sum up, for the transitive model from leader-follower to bargaining, retailer has a higher incentive to carry over strategic inventories to alter period 2's negotiation outcome when her bargaining power is relatively low, namely $0 \le \alpha < 1/2$, and only manages to do so under a relatively low storage cost $h \le \frac{(1-\alpha)(1-2\alpha)a}{3-2\alpha}$, as a consequence of supplier's pricing strategy analyzed as above.

For $0 \le \alpha < 1/2$, the optimal w^{1*} varies among $\frac{a}{2}$, $(1-\alpha)a-h$, $\frac{2(1-\alpha)a}{3-2\alpha}$ when h falls into respective domains ranging from high to low; these w^{1*}'s are verifiably in an ascending order. Note that as the wholesale price goes up, the first period sales quantity shrinks and moves farther away from the static leader-follower optimal as well as the first-best optimal, which indicates a constantly cut-down channel profit on period 1's sales. The appearance of strategic inventories would not reverse period 2's channel profit, which, being a product of cooperation, remains the first-best optimal; however, it will harm the total channel profit for the storage is charged.

When strategic inventories are feasible, double marginalization in period 1 is worsened as compared to static leader-follower setting, regardless of whether inventories are held or not. Recall that $1/2 \le \alpha \le 1$ implies $(1-\alpha)a - h < \frac{2(1-\alpha)a}{3-2\alpha} < \frac{a}{2}$. Whenever a higher w^1 is charged, q^1 is pushed farther away from $q^{fb} = \frac{a}{2b}$. On top of that, the channel profit can go even lower when additional inventory holding cost is incurred.

While retailer realizes she could take advantage of the strategic inventories to induce a Cournot-like supply-side competition in period 2 in order to virtually enlarge her bargaining power (and she may as well do so), supplier feels threatened, and the double marginalization is intensified whenever strategic inventories are feasible. The chain becomes less coordinated, which accounts for a diminished channel profit.

Chapter 3

Models and Analysis of Double Supply Chains with Horizontal Competition, Vertical Competitions and Cooperations

In the previous chapter with the focus on single supply chain of one product, by implementing bargaining framework partially or completely, we have seen existence of strategic inventories under certain settings, and absence in the others, due to various reasons with similar or opposite roles that inventories may play. Over the entire time horizon, if supplier and retailer stick to the leader-follower game, strategic inventories can lift the sales quantities towards the first-best optimal and increase channel's surplus, and are thus, carried; if supplier and retailer are in full cooperation, first-best sales quantities are always procured and retailed, and no inventories are necessary. When the supply chain is in a transition phase from a non-cooperative game to cooperative, or vice versa, the strategic role of inventories, the ability and consequences of carrying inventories can vary remarkably. When supply chain converts from negotiation to leader-follower, inventories continue to play a role to stimulate procurement and retailing from which both entities mutually benefit. In contrast, when supply chain is in transit from leader-follower contracting to bargaining, the inventories are anticipated to implicitly enlarge retailer's bargaining power, thus, intensify the horizontal competition and worsen the double marginalization effect.

Having seen a great deal on optimal contracts of single supply chain, we are motivated to see by introducing horizontal competition into the system, how much will the supply chain performance and management deviate, and what will trigger strategic inventories. Moreover, are there other roles of inventories to be discovered? With these questions in mind, we set up the following models and will present our results, analysis and interesting findings in this section.

Closely following Chapter 2, we now study a collection of models with similar settings applied to two chains, between which the horizontal (inter-chain) quantity-setting competition is introduced. These two chains, each consisting of a single supplier and a single retailer, conducting sales of two substitutable products with substitute intensity indexed by θ under linear demand curves, full information and no uncertainty. We restrain our study on a symmetric case with the market clearing price for product i given by $p(q_i, q_j) = a - bq_i - \theta bq_j$, where q_i, q_j are the sales quantities of product $i \neq j \in 1, 2$ respectively. All other parameters remain the same as in Chapter 2.

We first recap that, with horizontal competition, the first-best outcome for double-chain model is a sales quantity of $q^{fb^2} = \frac{a}{2(1+\theta)b}$ for both chains in each period. (Here the additional superscript 2 is to respect double-chain models to differentiate from quantities for single chain models.) Managerially, such a pair of sales quantities is only achieved if two supply chains form a cartel with the first-best channel profit of $\Pi_C^{fb^2} = \frac{a^2}{4(1+\theta)b}$ achieved for either chain. In static (single-period) double-chain leader-follower model, as well as the two-period commitment model (where no inventory is allowed), the optimal sales quantity in equilibrium is $q^{c^2*} = \frac{2a}{(4-\theta)(2+\theta)b}$ and the corresponding channel profit is $\Pi_C^{c^2*} = \frac{2(6-\theta^2)a^2}{(4-\theta)^2(2+\theta)^2 b}$. A relation of $q^{c^2*} < q^{fb^2}$ can be shown with slight work, and $\Pi_C^{c^2*} < \Pi_C^{fb^2}$ immediately follows. Such a gap in channel profit arises from both the vertical competition, where double marginalization is a major liability, as well as the horizontal competition that the quantity-setting game compels the sales quantities a pair of strategic complements. These two types of competition seem to jointly confine two chains' sales quantities further down below the first-best outcome, fertilize the soil to plant seeds of strategic inventories and convey us valid reasons to investigate double-chain models. On top, when intra-chain bargainings are implemented, vertical competition is cut out, which may provide us with a better vision for an anticipated battle between strategic inventories versus the horizontal competition. Hence, we proceed our work as such.

3.1 Models and Results

Consider a two-period model of two parallel supply chains $i = 1, 2$, each consisting of a single manufacturer and a single retailer conducting wholesale and retail business of a single product. The two goods are substitutes to each other with both unit production costs normalized to zero. Under a pair of sales quantities (q_1, q_2), the market-clearing price for product 1 is given by $p(q_1, q_2)$ and for product 2 is given by $p(q_2, q_1)$, where $p(x, y) := a - bx - \theta by$ and $\theta \in [0, 1]$ is the substitute intensity. Throughout the section of double-chain models, we use p_i^t to denote the market-clearing price for product i in period t, $i = 1, 2$, $t = 1, 2$ if no confusion aries.

3.1.1 Dynamic Leader-follower

Under a dynamic leader-follower model, at the beginning of period $t = 1, 2$, supplier $i = 1, 2$ simultaneously quote a wholesale price w_i^t per unit which instantaneously appear as public information to both chains. Observing these wholesale prices, retailers respond at the same time with a purchase quantity Q_i^t. In period 1 upon procurement, retailers further simultaneously determine their sales quantities $q_i^1 \leq Q_I^1$ and hold the rest $I_i = Q_i^1 - q_i^1$ as inventories with a unit inventory holding cost of h incurred. In period 2 after placing the order, retailers sell all the goods on hand of quantities of $q_i^2 = Q_i^2 + I_i$ to the market. The objective of the manufacturers and the retailers is to maximize their profits respectively. The two-period game is modelled as follows.

Period 2: Recall that in period 2, the strategic inventories I_1 and I_2 carried from period 1 have been known to both chains. Provided a pair of wholesale prices (w_1^2, w_2^2) quoted by suppliers, retailers independently determine their sales quantities q_1^2 and q_2^2 on account of each other's response. Each retailer's individual will is to maximize her profit if the other retailer's sales quantity is given. For example, retailer 1 would like to solve

$$\max_{q_1^2 \geq I_1} \left\{ \Pi_{R_1}^2 := p_1^2 q_1^2 - w_1^2 (q_1^2 - I_1) \right\}.$$

The competing effect between two retailers establishes an equilibrium on (q_1^2, q_2^2) using the optimal individual responses. Solving this equilibrium results in retailers' joint response $(q_1^{1*}(w_1^2, w_2^2), q_2^{1*}(w_1^2, w_2^2))$ conditional on the wholesale prices quoted by suppliers.

Knowing retailers' response, suppliers needs to independently determine their wholesale prices also taking account of the other supplier's action. Their individual will is similar to retailer, to maximize the profit if the other supplier's wholesale price is given. Supplier 1's perspective gives:

$$\max_{w_1^2 \geq 0} \left\{ \Pi_{S_1}^2 := w_1^2 (q_1^{2*}(w_1^2, w_2^2) - I_1) \right\}.$$

An equilibrium is then established on the pair of wholesale prices (w_1^2, w_2^2), leading to suppliers' joint decision $(w_1^{2*}(I_1, I_2), w_2^{2*}(I_1, I_2))$.

Period 1: The strategy to make decision is similar to that in period 1 but the profit to maximize for either supplier or retailer is the total value over two periods. Taking account to the other retailer's/supplier's action leads to an equilibrium and solving this equilibrium results in the retailers'/suppliers' joint response. The reader may refer to the complicate final result and the detailed derivation in Appendix B.1.

3.1.2 Cooperation with One-time Bargaining

For chain i, supplier and retailer bilaterally bargain over the wholesale prices w_i^t and sales quantities q_i^t for both periods $t = 1, 2$ as well as I_i, the amount of inventories carried over between periods, all in one shot at the beginning of period 1, in order to maximize their joint utility established in a generalized Nash bargaining game with retailer's bargaining power vis-a-vis supplier indexed by $\alpha \in [0, 1]$. Storage for each unit of inventories is charged h per period. A failure in negotiation leads to a zero-profit for both entities. All the other parameters follow from Section 3.1.1. Let $\Pi_{S_1}^t$ and $\Pi_{R_i}^t$, respectively, denote the profit function of supplier i and retail i in period t, $i = 1, 2$, $t = 1, 2$, so that $\Pi_{S_i} = \Pi_{S_i}^1 + \Pi_{S_i}^2$ and $\Pi_{R_i} = \Pi_{R_i}^1 + \Pi_{R_i}^2$. The two-period game is then modelled as follows.

Each chain has to independently make a decision on account of the possible action of the other chain. The decision for chain i includes wholesales prices w_i^1, w_i^2, sales quantities q_i^1, q_i^2 and inventory quantity I_i, $i = 1, 2$. Existence of two chains makes the joint decision finally becomes an equilibrium point on the individual best response of each provided the other chain's action is known. Such an individual best response of chain i ($i = 1, 2$) is to maximize its total utility function as

$$\max_{(w_i^1, w_i^2, q_i^1, I_i) \geq 0,\, q_i^2 \geq I_i} \left\{ (\Pi_{S_i} - D_{S_i})^{1-\alpha} (\Pi_{R_i} - D_{R_i})^{\alpha} \mid \Pi_{S_i} \geq D_{S_i}, \Pi_{R_i} \geq D_{R_i} \right\},$$

where $\Pi_{S_i} = \Pi_{S_i}^1 + \Pi_{S_i}^2$ and $\Pi_{R_i} = \Pi_{R_i}^1 + \Pi_{R_i}^2$ with

$$\Pi_{S_i}^1 = w_i^1(q_i^1 + I_i), \quad \Pi_{R_i}^1 = p_i^1 q_i^1 - w_i^1(q_i^1 + I_i) - hI_i, \tag{3.1}$$

$$\Pi_{S_i}^2 = w_i^2(q_i^2 - I_i), \quad \Pi_{R_i}^2 = p_i^2 q_i^2 - w_1^2(q_i^2 - I_i), \tag{3.2}$$

$$D_{S_i} = D_{R_i} = 0. \tag{3.3}$$

We end up with the optimal contract as

$$\left(q_1^{1*}, q_2^{1*}\right) = \left(q_1^{2*}, q_2^{2*}\right) = \left(\frac{a}{(2+\theta)b}, \frac{a}{(2+\theta)b}\right), \quad (I_1^*, I_2^*) = (0, 0).$$

The correspondingly channel profits of two chains are

$$(\Pi_{C_1}, \Pi_{C_2}) = \left(\frac{2a^2}{b(2+\theta)^2}, \frac{2a^2}{b(2+\theta)^2}\right).$$

See the detailed derivation in Appendix B.2.

3.1.3 Cooperation with Two-time Bargaining

The double-chain two-time bargaining model follows exactly from the single-chain two-time bargaining setting in Section 2.1.2 with the only exception that the market clearing price is modified. The two-period game is modelled as follows.

Period 2: Each chain has to independently make a decision on account of the possible action of the other chain. The decision strategy is the same to that of double-chain one-time bargaining in Section 3.1.2 but replacing the total utility function with the utility function in period 2. Therefore, given the inventory quantities (I_1, I_2) carried from period 1, the joint decision of two chains is an equilibrium point on the individual best response, which comes from maximizing the total utility function as

$$\max_{(w_i^1, w_i^2, q_i^1, I_i)\geq 0,\, q_i^2 \geq I_i} \left\{ (\Pi_{S_i} - D_{S_i})^{1-\alpha} (\Pi_{R_i} - D_{R_i})^{\alpha} \mid \Pi_{S_i} \geq D_{S_i}, \Pi_{R_i} \geq D_{R_i} \right\},$$

Period 1: This period is also conceptually the same to the double-chain one-time bargaining in Section 3.1.2. But the total utility function in the maximization for deriving each chain's individual best response for establishing the equilibrium is differently. We simply assume that the disagreement point is $(0, 0)$, which means that once the negotiation fails in Period 1, the chain ceases the operation till the end of Period 2. (Indeed, the result remains the same for any constant disagreement point.) Taking chain 1 for example, the maximization of its total utility is give by

$$\max_{(w_1^1, q_1^1, I_1)\geq 0} \left\{ \left(\Pi_{S_1} - D_{S_1}\right)^{1-\alpha} \left(\Pi_{R_1} - D_{R_1}\right)^{\alpha} \mid \Pi_{S_1} \geq D_{S_1}, \Pi_{R_1} \geq D_{R_1} \right\}, \tag{3.4}$$

where $D_{S_1} = 0$, $D_{R_1} = 0$ and

$$\Pi_{S_1} := \Pi_{S_1}^1 + \Pi_{S_1}^{2*}(I_1, I_2), \quad \Pi_{R_1} := \Pi_{R_1}^1 + \Pi_{R_1}^{2*}(I_1, I_2).$$

with $\Pi_{S_1}^1$, Π_{R_1} taking the forms as in (3.2) and $\Pi_{S_1}^{2*}(I_1, I_2), \Pi_{R_1}^{2*}(I_1, I_2)$ coming from the optimal solution conditional on (I_1, I_2) in period 2. The detailed derivation can be found in Appendix B.3. The optimal contract is

$$(q_1^{1*}, q_2^{1*}) = \left(\frac{a}{(2+\theta)b}, \frac{a}{(2+\theta)b} \right),$$

$$(I_1^*, I_2^*) = \begin{cases} (0,0) & \text{if } h > \bar{h}, \\ (0,0), \left(0, \dfrac{(2-\theta)a - 2h}{2(2-\theta^2)b}\right), \left(\dfrac{(2-\theta)a - 2h}{2(2-\theta^2)b}, 0\right) & \text{if } \theta > 0, h = \bar{h} \\ \left(0, \dfrac{(2-\theta)a - 2h}{2(2-\theta^2)b}\right), \left(\dfrac{(2-\theta)a - 2h}{2(2-\theta^2)b}, 0\right) & \text{if } \theta > 0, h < \bar{h}, \end{cases}$$

where

$$\bar{h} := \frac{4-\theta^2-\sqrt{8(2-\theta^2)}}{2(2+\theta)}a.$$

Correspondingly, the channel profits of two chains take the form (with the same order as above)

$$\left(\Pi^*_{C_1}, \Pi^*_{C_2}\right) = \begin{cases} (P_1, P_1) & \text{if } h > \bar{h}, \\ (P_1, P_1), (P_2, P_3), (P_3, P_2) & \text{if } \theta > 0, h = \bar{h}, \\ (P_2, P_3), (P_3, P_2) & \text{if } \theta > 0, h < \bar{h}, \end{cases}$$

where

$$P_1 = \frac{2a^2}{(2+\theta)^2 b}, \quad P_2 = \frac{a^2}{(2+\theta)^2 b} + \frac{((4-2\theta-\theta^2)a-2\theta h)^2}{8(2-\theta^2)b},$$
$$P_3 = \frac{a^2}{(2+\theta)^2 b} + \frac{((2-\theta)a-2h)^2}{8(2-\theta^2)b}.$$

3.1.4 Leader-follower + Bargaining

With a modification in the price function, the transitive model almost duplicates the setting in Section 2.1.3. Under this framework, period 2 is conceptually the same as period 2 under cooperation with two-time bargaining in Section 3.1.3; while period 1 is conceptually the same as period 1 under dynamic leader-follower model in Section 3.1.1. With detailed discussions in Appendix B.5, we end up with the optimal contract taking the form

- If $0 \leq \alpha < \frac{4-2\theta+\theta^2}{2(4-\theta)}$, then

$$I_1^* = I_2^* = \begin{cases} \dfrac{\bar{h}_1 - h}{2(1-\alpha)b} & \text{if } 0 \leq h \leq \bar{h}_1, \\ 0 & \text{if } \bar{h}_1 < h \leq \bar{h}_2, \\ 0 & \text{if } h > \bar{h}_2, \end{cases}$$

$$p_1^{1*} = p_2^{1*} = \begin{cases} \dfrac{2(1-\alpha)\theta + (4-\theta)^2}{(2+\theta)(2(1-\alpha)(4-\theta) + (4-\theta^2))b} & \text{if } 0 \leq h \leq \bar{h}_1, \\ \dfrac{(\alpha+\theta)a + h}{(2+\theta)^2 b} & \text{if } \bar{h}_1 < h \leq \bar{h}_2, \\ \dfrac{2a}{(2+\theta)(4-\theta)b} & \text{if } h > \bar{h}_2, \end{cases}$$

$$\Pi_{C_1}^* = \Pi_{C_2}^* = \begin{cases} \dfrac{2a^2}{(2+\theta)^2 b} - \dfrac{(2-\theta)(6+\theta)a^2}{(2+\theta)^2(4-\theta)^2 b} + \Delta(\bar{h}_2 - \bar{h}_1) - \dfrac{h(\bar{h}_1 - h)}{2(1-\alpha)b} & \text{if } 0 \leq h \leq \bar{h}_1, \\ \dfrac{2a^2}{(2+\theta)^2 b} - \dfrac{(2-\theta)(6+\theta)a^2}{(2+\theta)^2(4-\theta)^2 b} + \Delta(\bar{h}_2 - h) & \text{if } \bar{h}_2 < h \leq \bar{h}_2, \\ \dfrac{2a^2}{(2+\theta)^2 b} - \dfrac{(2-\theta)(6+\theta)a^2}{(2+\theta)^2(4-\theta)^2 b} & \text{if } h > \bar{h}_2, \end{cases}$$

where

$$\bar{h}_1 := \frac{2(1-\alpha)\left(2(1-\alpha)(4-\theta) - (4-\theta^2)\right)}{(2+\theta)\left(2(1-\alpha)(4-\theta) + (4-\theta^2)\right)} a \quad \text{and} \quad \bar{h}_2 := \frac{2(1-\alpha)(4-\theta) - (4-\theta^2)}{(2+\theta)(4-\theta)} a,$$

and

$$\Delta(t) := \frac{1+\theta}{2+\theta}\left(t^2 - \frac{4+\theta^2}{2(1+\theta)(4-\theta)} t\right).$$

- If $\alpha \geq \frac{4-2\theta+\theta^2}{2(4-\theta)}$, then

$$I_1^* = I_2^* = \frac{(\bar{h}_2 - h)^+}{2(1-\alpha)b} \quad \forall\, h \geq 0,$$

$$p_1^{1*} = p_2^{1*} = \frac{2a}{(2+\theta)(4-\theta)b} \quad \forall\, h \geq 0,$$

$$\Pi_{C_1}^* = \Pi_{C_2}^* = \frac{2a^2}{(2+\theta)^2 b} - \frac{(2-\theta)(6+\theta)a^2}{(2+\theta)^2(4-\theta)^2 b} - \frac{h(\bar{h}_2 - h)^+}{2(1-\alpha)b} \quad \forall\, h \geq 0.$$

3.1.5 Bargaining + Leader-follower

With a modification in the price function, the transitive model almost duplicates the setting in Section 2.1.4. Under this framework, period 2 is conceptually the same as period 2 under dynamic leader-follower model in Section 3.1.1; while period 1 is conceptually the same as period 1 under cooperation with two-time bargaining Section 3.1.3. With detailed discussions in Appendix B.5, we end up with the optimal contract taking the form

$$q_1^{1*} = q_2^{1*} = \frac{a}{(2+\theta)b},$$
$$I_1^* = I_2^* = \frac{(32 - 8\theta^2 + \theta^4)a - (2+\theta)(4+\theta)(4-\theta)^2 h}{(4-\theta^2)(16+8\theta-4\theta^2-\theta^3)b}.$$

3.2 Comparison and Analysis

In double-chain dynamic model in which supplier and retailer play leader-follower games across periods, concurrence of the horizontal and vertical competitions, on one hand, does trigger the holding of strategic inventories for both chains, but also complicates the analysis if we want to separate the effect of either and address the corresponding strategic roles of inventories in response independently. Through an analysis of direct and indirect economic effect, we state our findings in the following theorem.

Theorem 3.1. *In the optimal contract in equilibrium for double-chain dynamic model,*

i) inventories are carried by both retailers, each chain's period 2 sales quantity is pushed up, and the total channel profit is boosted in comparison with the optimal outcome in the static double-chain model.

ii) One strategic role of the inventories is to soothe the vertical competition by impairing supplier's monopolistic power in period 2, which is a direct duplicate of results from the single-chain dynamic model.

iii) Inventories for competitive chains are shown to be strategic substitutes to each other, unlike the sales quantities which are strategic complement. Since the inventories partially constitute sales quantities in period 2, they effectively soften the horizontal competition.

To elaborate further how strategic inventories ease the horizontal competition, we first apply a direct and indirect effect analysis on respective metrics and obtain

$$\frac{\mathrm{d}q_1^2}{\mathrm{d}q_2^2} = -\frac{\theta}{2} < 0, \qquad \frac{\mathrm{d}I_1}{\mathrm{d}I_2} = \frac{\partial I_1}{\partial w_1^1}\frac{\partial w_1^1}{\partial I_2} + \frac{\partial I_1}{\partial w_2^1}\frac{\partial w_2^1}{\partial I_2} > 0,$$

where the second inequality comes from the fact that $\frac{\partial I_1}{\partial w_1^1} < 0$, $\frac{\partial w_1^1}{\partial I_2} = 0$, $\frac{\partial I_1}{\partial w_2^1} > 0$ and $\frac{\partial w_2^1}{\partial I_2} > 0$. This means that I_1, I_2 are strategic substitutes, meaning an increment in I_2 will increase I_1. In contrast, q_1^2, q_2^2 are strategic complements, meaning q_1^2 is going down when q_2^2 goes up.

Being strategic complements to each other keeps the sales quantities away from the first-best outcome. Now that a pre-procurement of inventories is allowed, inventories serve as stimulant to each other to purchase more quantities and push the sales towards to the first-best optimal. This analysis fully focuses on, contributes to and addresses issues with respect to horizontal competition, and uncovers an untouched strategic role of inventories.

Next, we will compare bargaining framework with the static leader-follower as well as first-best outcome in double-chain models. Recall that in single-chain models, bargaining framework is equivalent to establish a centralized system and achieves the first-best solutions. We will see otherwise in next theorem.

Theorem 3.2. *i) The optimal contract in equilibrium for one-time bargaining double-chain model mimics in each period the outcome of static (single-period) bargaining model. In each period, both chains purchase and retail an amount of* $q^{b^2*} = \frac{a}{(2+\theta)b}$ *and reach a channel profit of* $\Pi_C^{b^2*} = \frac{a}{(2+\theta)^2 b}$.

ii) To compare among the optimal solutions for commitment contracting, one-time bargaining, and the first-best outcome, we have the following relations:

$$\begin{aligned}
&q^{c^2*} < q^{fb^2} < q^{b^2*},\\
&\Pi_C^{c^2*} < \Pi_C^{fb^2}, \quad \Pi_C^{b^2*} < \Pi_C^{fb^2},\\
&\Pi_C^{c^2*} \leq \Pi_C^{b^2*} \quad \textit{if } 0 \leq \theta \leq 2/3,\\
&\Pi_C^{b^2*} < \Pi_C^{c^2*} \quad \textit{if } 2/3 < \theta \leq 1.
\end{aligned}$$

Now that we realize horizontal competition changes the whole story and the dominance of bargaining framework is out, we are motivated to explore on whether or not strategic inventories will exist and can be a remedy to supply chain coordination, especially when horizontal competition is intense. It turns out that when two-time bargaining is conducted, asymmetric equilibria can exist with one chain indeed holding proper amount of inventories, and inventories have showcased other duties too.

Theorem 3.3. *i) In optimal contracts for model in Section 3.1.3, both chains copy the optimal contract for static bargaining in period 1.*

ii) Asymmetric equilibria $(I_1^*, I_2^*) = \left(0, \frac{(2-\theta)a-2h}{2(2-\theta^2)b}\right), \left(\frac{(2-\theta)a-2h}{2(2-\theta^2)b}, 0\right)$ *exist if* $\theta > 0, h \leq \bar{h} = \frac{4-\theta^2-\sqrt{8(2-\theta^2)}}{2(2+\theta)}a$.

iii) For the equilibrium $(I_1^*, I_2^*) = \left(0, \frac{(2-\theta)a-2h}{2(2-\theta^2)b}\right)$ *(while the other is simply a mirror image),* $\left(\Pi_{C_1}^{2*}, \Pi_{C_2}^{2*}\right) = \left(\frac{((4-2\theta-\theta^2)a-2\theta h)^2}{8(2-\theta^2)b}, \frac{((2-\theta)a-2h)^2}{8(2-\theta^2)b}\right)$. *Moreover,* $\Pi_{C_1}^{2*} > \Pi_{C_2}^{2*} > \Pi_C^{b^2*} = \frac{a^2}{(2+\theta)^2 b}$, *i.e. both chains are better-off when one chain carries the inventories.*

Inventories in the optimal contract essentially can be seen as a signalling tool or one chain's commitment to the other to sustain a collusive behavior, which does improve the system coordination.

First, we recognize that at the equilibrium point, one chain chooses to hold strategic inventories and the other not, and given this strategy profile, both chains benefit, which confirms that inventories have played a strategic role.

What seems a bit counter-intuitive is that, chain 1 who holds no inventories will end up with a larger profit than chain 2; yet, chain 2 is still incentivized to carry over inventories as both chains' profits are strictly better-off as compared to the case with no inventories allowed. To understand the inherent reason, we trace back to the previous theorem and acknowledge that when intra-chain bargaining occurs, both chains tend to procure too much that the sales quantities overflow/ exceed the first-best outcome. The equilibrium points suggest that when inventories are feasible (namely when holding cost is reasonably fair), chain 2 chooses to carry over inventories strategically to signal to chain 1 that they themselves will commit to a sales quantity up to the inventories level in future period, inducing chain 1 to a lower level sales correspondingly. This weakens the quantity competition between two chains and results in an increase in both chains' channel profits.

To sum up, an interesting finding appears in the double-chain studies that two-time bargaining can be greatly different from one-time bargaining when horizontal competition exists. Very much unlike the single-chain models, bargainings dominance to strategic inventories is changed as it can sometimes intensify the quantity competition in comparison with the leader-follower game by pushing up the sales quantity, resulting in a lower channel profits for both chains. On the contrary, the ability in carrying inventories can be used strategically by one chain as a commitment to the other to ease up the quantity-setting competition and to essentially sustain a collusive behavior, hence, a win-win situation is anticipated.

We will continue to facilitate a brief comparison between the single-chain (in Section 2.1.4) and double-chain (in Section 3.1.4) leader-follower + bargaining models results analytically.

Theorem 3.4. *i) A similar pattern to single-chain model in the optimal solution is observed for double-chain, and only symmetric equilibrium exists.*

ii) Strategic inventories continue to harm the channel profits by worsening the horizontal competition.

iii) The quoted period-1 wholesale price at equilibrium for double-chain is universally lower than that for single-chain model.

iv) The optimal inventory level for double-chain is lower than that of single-chain, too, if inventories are indeed carried.

Chapter 4

Conclusions and Future Research

Inspired by a simple stylized dynamic model, we have delivered our concerns with comparison and contrast in bargaining framework and the effect of strategic inventories, raised questions in the existence of inventories in optimal contracts as well as the corresponding supply chain performance, and further addressed issues regarding the change of supply chain coordination under bargaining when horizontal competition is introduced into the system. We use models to incorporate one or more of the following characteristics — competition in vertical control, cooperation via bilateral bargaining, horizontal competition between retailers sourcing from independent suppliers — and deduce the respective optimal solutions, based on which some preliminary perceptions on various roles of strategic inventories are established, while further managerial insights still await to be discovered. We wrap up our current studies by a brief summary on the explanatory roles of strategic inventories in respective models.

When single-chain is concerned, for scenarios when competition exists in vertical controls, strategic inventories can be used to break suppliers monopoly power and reduce the channel profit loss due to double marginalization effect. Retailer can also be incentivized to hold inventories to in effect enhance her bargaining power when negotiation is to take place. However, if cooperation occurs throughout the entire time horizon, inventories are not held in optimal contract due to a drain of additional holding from the channel profit. On the other hand, when the chain is in a transition phase, supplier intends to avoid such a threat, and the vertical competition is actually intensified. To consider interactions between two parallel chains, inventories continue to play strategic roles in vertical controls; on top of that, other uses are speculated too. Proven to be strategic substitutes to each other, strategic inventories carried by competitive chains partially constitute their respective sales quantities, and the strategic complementarity between sales quantities are thus partially replaced. Consequently, larger sales quantities are realized, the gap to first-best optimal is bridged, and horizontal competition is softened with both chains mutually benefitted. Lastly, inventories are used as a commitment tool of one chain to the other to avoid concurrence of large sales quantities when two-time intra-chain bargaining framework is adopted. Under a decision of holding inventories beforehand, one chain is to substantially commit to a pre-determined sales quantity, in order to sustain the collusive behavior to induce the system to approach the first-best outcome.

The naturally perceived association between strategic inventory and the bargaining power is one of the fundamental motivations that we initiated this study. We did not incorporate the strategic inventory in the bargaining power mainly because we, in the first place, wanted to start by investigating to which level is bargaining able to replace the strategic inventory and whether or not more functionalities are covered. Another reason we did not implement the idea as the examiner suggested is that, currently, given the limited understanding of the role that strategic inventory plays, it is hard to identify the other factors implicitly in the bargaining power. To explore it more toward this direction is also something that has interested us and can be expected in the future studies.

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Appendices

A Single-chain Models

In this section, we will analyse the five two-period single-chain models described in Section 2.1 and end up with explicit expressions of trading terms and profits.

A.1 One-time Bargaining

In the single-chain model of cooperation with one-time bargaining, the two-period single-chain game is modelled as follows.

$$\max_{(w^1,w^2,q^1,I)\geq 0,\, q^2\geq I} \left\{(\Pi_S - D_S)^{1-\alpha}(\Pi_R - D_R)^{\alpha} \mid \Pi_S \geq D_S,\, \Pi_R \geq D_R\right\}. \tag{4.1}$$

where $D_S = D_R = 0$, and

$$\Pi_S = w^1(q^1 + I) + w^2(q^2 - I), \quad \Pi_R = p^1q^1 + p^2q^2 - w^1(q^1 + I) - w^2(q^2 - I) - hI.$$

It is not hard to derive from the KKT conditions that the optimal solution satisfies

$$\alpha(\Pi_S - D_S) = (1 - \alpha)(\Pi_R - D_R). \tag{4.2}$$

(The cases $\alpha = 0$ or $\alpha = 1$ can be achieved by the continuity argument.) Moreover, note that

$$(\Pi_S - D_S) + (\Pi_R - D_R) = p^1 q^1 + p^2 q^2 - hI.$$

Therefore, we can obtain that if $\left(w^{1*}, w^{2*}, q^{1*}, q^{2*}, I^*\right)$ is an optimal solution to (4.1), then we must have

$$\left(q^{1*}, q^{2*}, I^*\right) = \underset{(q^1, I) \geq 0,\, q^2 \geq I}{\arg\max} \left\{p^1 q^1 + p^2 q^2 - hI\right\}.$$

Benefiting from the separable structure of this optimization problem, we can easily have

$$q^{1*} = q^{2*} = \frac{a}{2b} \quad \text{and} \quad I^* = 0.$$

We further plug these values into (4.2) and then obtain $w_1^* + w_2^* = (1-\alpha)a$. Indeed, there exist infinitely many optimal solutions to (4.1).

A.2 Two-time Bargaining

In the model of cooperation with two-time bargaining, the two-period single-chain game is modelled as follows.

Period 2: Maximize the joint utility of profit in period 2.

$$\max_{w^2 \geq 0,\, q^2 \geq I} \left\{\left(\Pi_S^2 - D_S^2\right)^{1-\alpha} \left(\Pi_R^2 - D_R^2\right)^{\alpha} \mid \Pi_S^2 \geq D_S^2,\, \Pi_R^2 \geq D_R^2\right\}, \tag{4.3}$$

where

$$\Pi_S^2 = w^2(q^2 - I), \quad \Pi_R^2 = p^2 q^2 - w^2(q^2 - I), \quad D_S^2 = 0, \quad D_R^2 = p(I)I.$$

Note that

$$(\Pi_S^2 - D_S^2) + (\Pi_R^2 - D_R^2) = p^2 q^2 - p(I)I.$$

Using the same argument as in Appendix A.1, we have

$$\alpha(\Pi_S^2 - D_S^2) = (1-\alpha)(\Pi_R^2 - D_R^2), \tag{4.4}$$

and moreover,

$$q^{2*}(I) = \arg\max_{q^2 \geq I} \left\{p^2 q^2 - p(I)I\right\} = \frac{(a-2bI)^+}{2b} + I.$$

We then plug this into (4.4) and obtain

$$w^{2*}(I) = \frac{1-\alpha}{2}(a-2bI)^+.$$

Correspondingly, the profits of supplier and retailer under the optimal solution are

$$\Pi_S^{2*}(I) = \frac{1-\alpha}{4b}\left((a-2bI)^+\right)^2 \quad \text{and} \quad \Pi_R^{2*}(I) = \frac{\alpha}{4b}\left((a-2bI)^+\right) + p(I)I.$$

Period 1: Maximize the joint utility of profit over two periods.

$$\max_{(w^1,q^1,I)\geq 0} \left\{(\Pi_S - D_S)^{1-\alpha}(\Pi_R - D_R)^{\alpha} \mid \Pi_R \geq D_R, \Pi_S \geq D_S\right\},$$

where $D_S = D_R = 0$, and

$$\Pi_S = w^1(q^1 + I) + \Pi_S^{2*}(I), \qquad \Pi_R = p^1 q^1 - w^1(q^1+I) - hI + \Pi_R^{2*}(I).$$

Note a separable structure in term of $(\Pi_S - D_S) + (\Pi_R - D_R) = \Pi_C^q + \Pi_C^I$, where

$$\Pi_C^q = p^1 q^1 \quad \text{and} \quad \Pi_C^I = \frac{1-\alpha}{4b}\left((a-2bI)^+\right)^2 + \frac{\alpha}{4b}\left((a-2bI)^+\right) + p(I)I - hI.$$

Therefore, using the same argument as in Appendix A.1, we have

$$q^{1*} = \arg\max_{q^1\geq 0} \left\{\Pi_C^q\right\} \quad \text{and} \quad I^* = \arg\max_{I\geq 0} \left\{\Pi_C^I\right\}.$$

Direct calculation using the first-order optimality condition yields

$$q^{1*} = \frac{a}{2b}, \qquad I^* = 0, \quad \text{and} \quad w^{1*} = \frac{(1-\alpha)a}{2}.$$

We further plug I^* back into the expressions in period 2 and then obtain

$$w^{2*} = \frac{(1-\alpha)a}{2} \quad \text{and} \quad q^{2*} = \frac{a}{2b}.$$

Moreover, the channel profits in each period under this solution is

$$\Pi_C^{t*} = \frac{a^2}{4b}, \quad \Pi_S^{t*} = (1-\alpha)\Pi_C^{t*}, \quad \Pi_R^{t} = \alpha\Pi_C^{t*}, \quad t = 1, 2.$$

A.3 Bargaining + Leader-follower

This two-period single-chain game under the "bargaining + leader-follower" framework is modelled as follows.

Period 2: Presuming an inventory quantity I from period 1 and a wholesale price w^2 quoted by supplier, retailer determines the sales quantity q^2 by maximizing his profit as

$$\max_{q^2 \geq I} \left\{ p^2 q^2 - w^2(q^2 - I) \right\}.$$

It is easy to obtain that the optimal solution is

$$q^{2*}(w^2) = \frac{(a - 2^2 - 2bI)^+}{2b} + I.$$

Knowing the response curve of retailer, supplier determines the wholesale price w^2 by maximizing his profit as

$$\max_{w^2 \geq 0} \left\{ w^2(q^{2*}(w^2) - I) \right\}.$$

Direct calculation yields

$$w^{2*}(I) = \frac{(a - 2bI)^+}{2}.$$

Correspondingly, under this optimal solution, the profits of supplier and retailer are

$$\Pi_S^{2*}(I) = \frac{((a-2bI)^+)^2}{8b} \quad \text{and} \quad \Pi_R^{2*}(I) = \frac{a^2}{4b} - \frac{3((a-2bI)^+)^2}{16b}.$$

Period 1: Suppler and retailer jointly determine the wholesale price w^1, the sales quantity q^1 and the inventory quantity I, aiming to maximize the utility of profit over two periods defined as follows:

$$\max_{(q^1,I,w^1)\geq 0} \left\{(\Pi_S - D_S)^{1-\alpha}(\Pi_R - D_R)^{\alpha} \mid \Pi_S \geq D_S,\, \Pi_R \geq D_R\right\}$$

where

$$\Pi_S = w^1(q^1+I) + \Pi_S^{2*}(I), \qquad \Pi_R = p^1q^1 - w^1(q^1+I) - hI + \Pi_R^{2*}(I),$$

and the disagreement point (D_R, D_S) is defined as the profits that retailer and supplier could achieve when the cooperation fails. Taking into account of the leader and follower's roles, we credit the optimal profits in the static leader-follower game to the disagreement point respectively. To be more specific,

$$D_R = \frac{a^2}{16b} \quad \text{and} \quad D_S = \frac{a^2}{8b}.$$

Note a separable structure in terms of $(\Pi_S - D_S) + (\Pi_R - D_R) = \Pi_C^q + \Pi_C^I$, where

$$\Pi_C^q = p^1q^1 \quad \text{and} \quad \Pi_C^I = \frac{a^2}{16b} - \frac{((a-2bI)^+)^2}{16b} - hI.$$

Using the same argument as in Appendix A.1, we have

$$q^{1*} = \arg\max_{q^1\geq 0} \,\{\Pi_C^q\} \quad \text{and} \quad I^* = \arg\max_{I\geq 0} \,\left\{\Pi_C^I\right\}.$$

Direct calculation yields

$$q^{1*} = \frac{a}{2b}, \quad I^* = \frac{a-4h}{2b}, \quad w^{1*} = \frac{(7-5\alpha^2)a^2 - 8(1-\alpha)ah - 16(1+\alpha)h^2}{16(a-2h)}.$$

Correspondingly, the total profits for channel, supplier and retailer are

$$\Pi_C^* = \frac{a^2-ah+2h^2}{2b},\ \Pi_S^* = (1-\alpha)\left(\frac{5a^2}{16b} - \frac{ah}{2b} + \frac{h^2}{b}\right) + \frac{a^2}{8b},\ \Pi_R^* = \alpha\left(\frac{5a^2}{16b} - \frac{ah}{2b} + \frac{h^2}{b}\right) + \frac{a^2}{16b}.$$

Plugging I^* back into the expressions in period 2, we further have

$$w^{2*} = 2h, \quad q^{2*} = \frac{a-2h}{2b}, \quad \Pi_C^{2*} = \frac{a^2}{4b} - \frac{h^2}{b}, \quad \Pi_S^{2*} = \frac{2h^2}{b}, \quad \Pi_R^{2*} = \frac{a^2}{4b} - \frac{3h^2}{b}.$$

A.4 Leader-follower + Bargaining

The two-period single-chain game under the "leader-follower + Bargaining" framework is modelled as follows.

Period 2: The discussion is exactly the same as that of period 2 under cooperation with Two-time Bargaining in Appendix A.2.

Period 1: Given a wholesale price w^1 quoted by supplier, retailer determines the sales quantity q^1 and the inventory quantity I by maximizing his total profit over two periods as

$$\max_{(q^1,I)\geq 0} \left\{\Pi_R := \Pi_R^1 + \Pi_R^{2*}(I)\right\},$$

where Π_R^1 takes the form in (2.1) and $\Pi_R^{2*}(I)$ takes the form in Appendix A.2. More specifically,

$$\Pi_R^1 = p^1q^1 - w^1(q^1+I) - hI \quad \text{and} \quad \Pi_R^{2*}(I) = \frac{\alpha}{4b}\left((a-2bI)^+\right) + p(I)I.$$

Spot that Π_R is separable in q^1 and I. Using the first-order optimality condition yields the unique solutions

$$q^{1*}(w^1) = \frac{a-w^1}{2b} \quad \text{and} \quad I^*(w^1) = \left(\frac{a}{2b} - \frac{h+w^1}{(1-\alpha)2b}\right)^+.$$

Reflecting in supplier's profit function, we backwards solve for

$$\max_{w^1\geq 0} \left\{\Pi_S := \Pi_S^1\left(q^{1*}(w^1), I^*(w^1)\right) + \Pi_S^{2*}\left(I^*(w^1)\right)\right\},$$

where Π_S^1 takes the form in (2.1) and $\Pi_R^{2*}(I)$ takes the form in Appendix A.2 with $I = I^*(w^1)$, $q^1 = q^{1*}(w^1)$. More specifically,

$$\begin{aligned}\Pi_S &= w^1(q^{1*}(w^1) + I^*(w^1)) + \frac{1-\alpha}{4b}\left((a-2bI^*(w^1))^+\right)^2 \\ &= \begin{cases} \dfrac{(1-\alpha)a^2}{(3-2\alpha)b} + \dfrac{h^2}{4(1-\alpha)b} - \dfrac{3-2\alpha}{4(1-\alpha)b}\left(w^1 - \dfrac{2(1-\alpha)a}{3-2\alpha}\right)^2 & \text{if } w^1 \leq (1-\alpha)a-h, \\[2ex] \dfrac{(3-2\alpha)a^2}{8b} - \dfrac{1}{2b}\left(w^1 - \dfrac{a}{2}\right)^2 & \text{if } w^1 > (1-\alpha)a-h. \end{cases}\end{aligned}$$

Note that Π_S is a piecewise continuous function with a break point $w_m^1 = (1 - \alpha)a - h$. Let Π_S^l and Π_S^r denote the left and right subfunctions, respectively. It is obvious that the global maximizer of Π_S^l and the global maximizer of Π_S^r are, respectively,

$$w_l^1 := \frac{2(1-\alpha)a}{3-2\alpha} \quad \text{and} \quad w_r^1 := \frac{a}{2}.$$

Define

$$\bar{h}_1 := \frac{(1-\alpha)(1-2\alpha)a}{3-2\alpha} \quad \text{and} \quad \bar{h}_2 := \frac{(1-2\alpha)a}{2} \quad \left(\bar{h}_1 \le \bar{h}_2\right).$$

Then we have

- If $h \le \bar{h}_1$, then $w_l^1 \le w_m^1$, $w_r^1 \le w_m^1$, and thus $w^{1*} = w_l^1$.
- If $\bar{h}_1 < h \le \bar{h}_2$, then $w_l > w_m$, $w_r \le w_m$, and thus $\bar{w}^{1*} = w_m^1$.
- If $h > \bar{h}_2$, then $w_l > w_m$, $w_r > w_m$, and thus $\bar{w}^{1*} = w_r^1$.

Therefore, we conclude that

- If $0 \le \alpha < 1/2$, then

$$\begin{cases} w^{1*} = \dfrac{2(1-\alpha)}{3-2\alpha}a, \ I^* = \dfrac{(1-2\alpha)a}{2b} - \dfrac{h}{2(1-\alpha)b} \, (> 0) & \text{if } h \le \bar{h}_1, \\ w^{1*} = (1-\alpha)a - h, \ I^* = 0 & \text{if } \bar{h}_1 < h \le \bar{h}_2, \\ w^{1*} = \dfrac{a}{2}, \ I^* = 0 & \text{if } h > \bar{h}_2. \end{cases}$$

- If $1/2 \le \alpha \le 1$, then

$$w^{1*} = \frac{a}{2}, \quad I^* = 0 \quad \forall\, h \ge 0.$$

B Double-chain Models

In this section, we will analyse the five two-period double-chain models described in Section 3.1 and end up with explicit expressions of trading terms and profits.

B.1 Dynamic Leader-follower

In the model of dynamic leader-follower framework, the two-period double-chain game is modelled as follows.

Period 2: Recall that in period 2, the strategic inventories I_1 and I_2 carried from period 1 have been known to both chains. Provided a pair of wholesale prices (w_1^2, w_2^2) quoted by suppliers, retailers aim to determine their sales quantities q_1^2 and q_2^2 on account of each other's response. From retailer 1's perspective, given retailer 2's sales quantity q_2^2, she would like to maximize the profit $\Pi_{R_1}^2$, i.e.,

$$\max_{q_1^2 \geq I_1} \left\{ \Pi_{R_1}^2 := p_1^2 q_1^2 - w_1^2 (q_1^2 - I_1) \right\}.$$

The solution to this optimization problem is

$$\bar{q}_1^2 = \max \left\{ \frac{a - \theta b q_2^2 - w_1^2}{2b}, I_1 \right\}.$$

Meanwhile, from retailer 2's perspective, given retailer 1's sales quantity q_1^2, her decision $\bar{q}_2^2$ is symmetric, i.e.,

$$\bar{q}_2^2 = \max \left\{ \frac{a - \theta b q_1^2 - w_2^2}{2b}, I_2 \right\}.$$

The competing effect forces us to establish an equilibrium to find their joint response to the presumed wholesale prices. For simplicity of discussion, we assume that $\frac{a - \theta b q_2^{2*} - w_1^2}{2b} \geq I_1$ and $\frac{a - \theta b q_1^{2*} - w_2^2}{2b} \geq I_2$. This assumption can be explained as strategic inventories of retailers are bounded by a certain level. (Indeed, $I_1 \leq \frac{(2+\theta-\theta^2)a - (4+\theta-\theta^2)w_1 + 2w_2}{2(4-\theta^2)b}$ and $I_2 \leq \frac{(2+\theta-\theta^2)a - (4+\theta-\theta^2)w_2 + 2w_1}{2(4-\theta^2)b}$.) Under this assumption, by plugging the expression of $\bar{q}_2^2$ into the expression of $\bar{q}_1^2$, we obtain that

$$q_1^{2*}(w_1^2, w_2^2) = \frac{(2-\theta)a + \theta w_2 - 2w_1}{(4-\theta^2)b} \quad \text{and} \quad q_2^{2*}(w_1^2, w_2^2) = \frac{(2-\theta)a + \theta w_1 - 2w_2}{(4-\theta^2)b}.$$

We then help each supplier to determine her wholesale price, taking account of the other supplier's action. Take supplier 1 for example. Her will is similar to retailer's, i.e., to maximize the profit if supplier 2's wholesale price is given:

$$\max_{w_1^2 \geq 0} \left\{ \Pi_{S_1}^2 := w_1^2 \big(q_1^{2*}(w_1^2, w_2^2) - I_1 \big) \right\}.$$

The optimal solution to this maximization problem is

$$\bar{w}_1^2 = \frac{(2-\theta)a - (4-\theta^2)bI_1 + \theta w_2^2}{4}.$$

Supplier 2's perspective gives a symmetric decision $\bar{w}_2^2$ if supplier 1's wholesale price w_1^2 is given. An equilibrium is then established on the pair of wholesale prices (w_1^2, w_2^2). Solving this equilibrium results in the joint decision of suppliers as

$$w_1^{2*}(I_1,I_2) = \frac{4-\theta^2}{16-\theta^2}\left(\frac{4+\theta}{2+\theta}a - 4bI_1 - \theta bI_2\right), \quad w_2^{2*}(I_1,I_2) = \frac{4-\theta^2}{16-\theta^2}\left(\frac{4+\theta}{2+\theta}a - 4bI_2 - \theta bI_1\right).$$

Substituting (w_1^{2*}, w_2^{2*}) back, we have

$$\begin{aligned} q_1^{2*}(I_1, I_2) &= \frac{1}{(16-\theta^2)b}\left(\frac{2(4+\theta)}{2+\theta}a + (8-\theta^2)bI_1 - 2\theta bI_2\right), \\ p_1^{2*}(I_1, I_2) &= \frac{1}{16-\theta^2}\left(\frac{(4+\theta)(6-\theta^2)}{2+\theta}a - (8-3\theta^2)bI_1 - \theta(6-\theta^2)bI_2\right). \end{aligned}$$

Using the same argument leads to symmetric expressions of q_2^{2*}, p_2^{2*}. Then we can obtain the profits of suppliers and retailers under the optimal solution are

$$\begin{aligned} \Pi_{S_1}^{2*}(I_1, I_2) = \Pi_{S_2}^{2*}(I_1, I_2) &= \frac{2(4-\theta^2)}{(16-\theta^2)^2 b}\left(\frac{4+\theta}{2+\theta}a - 4bI_1 - \theta bI_2\right)^2, \\ \Pi_{R_1}^{2*}(I_1, I_2) = \Pi_{R_1}^{2*}(I_1, I_2) &= \frac{8-\theta^2}{8(16-\theta^2)b}\left(\frac{4+\theta}{2+\theta}a - \theta bI_2\right)^2 \\ &\quad - \frac{96-24\theta^2+\theta^4}{8(16-\theta^2)^2 b}\left(\frac{4+\theta}{2+\theta}a - 4bI_1 - \theta bI_2\right)^2 + \frac{\theta^2}{16-\theta^2}bI_1^2. \end{aligned}$$

Period 1: The strategy to make decision is similar to that in period 1 but the profit to maximize for either supplier or retailer is the total value over two periods. Presuming a pair of wholesale prices (w_1^1, w_2^1) quoted by suppliers, if retailer 2's decision is given, i.e., the inventory quantity I_2 and the sales quantity q_2^1, retailer 1 would like to maximize its total profit over two periods as

$$\max_{(q_1^1, I_1)\geq 0} \left\{\Pi_{R_1} := \Pi_{R_1}^1 + \Pi_{R_1}^{2*}(I_1, I_2)\right\},$$

where $\Pi_{R_1}^1 := p_1^1 q_1^1 - w_1^1(q_1^1 + I_1) - hI_1$ and $\Pi_{R_1}^{2*}(I_1, I_2)$ comes from period 2. This two-dimensional maximization problems can be separated into two one-dimensional maximization problems as

$$\max_{q_1^1\geq 0} \left\{\Pi_{R_1}^q := (a - bq_1^1 - \theta bq_2^1 - w_1^1)q_1^1\right\}, \quad \max_{I_1\geq 0} \left\{\Pi_{R_1}^I := -(w_1^1 + h)I_1 + \Pi_{R_1}^{2*}(I_1, I_2)\right\}.$$

By the first-order optimality condition, the solution to the first maximization problem is

$$\bar{q}_1^1 = \frac{(a - w_1^1 - \theta bq_2^1)^+}{2b},$$

and the solution to the second maximization problem is

$$\begin{aligned}\bar{I}_1 =& \big[(4+\theta)(96 - 24\theta^2 + \theta^4)a - \theta(2+\theta)(96 - 24\theta^2 + \theta^4)bI_2 - (2+\theta)(16-\theta^2)^2 w_1^1 \\ & - (2+\theta)(16-\theta^2)^2 h\big] \big/ \big[2(2+\theta)(192 - 64\theta^2 + 3\theta^4)b\big].\end{aligned}$$

The same argument can also be applied to retailer 2 to reach the symmetric expressions of the corresponding quantities $\bar{q}_2^1$ and $\bar{I}_2$. For simplicity, we assume that $a - w_1^1 - \theta bq_2^{1*} \geq 0$ and $a - w_1^1 - \theta bq_1^{1*} \geq 0$. Knowing the individual response of each retailer, we then solve the equilibrium to obtain retailer's joint response on account of the competing effect. Direct calculation yields that

$$\begin{aligned}q_1^{1*}(w_1^1, w_2^1) =& \frac{(2-\theta)a - 2w_1^1 + \theta w_2^1}{(2+\theta)(2-\theta)b}, \\ I_1^*(w_1^1, w_2^1) =& \big[(48 - 24\theta - 4\theta^2 + \theta^3)(96 - 24\theta^2 + \theta^4)a + \theta(4+\theta)(4-\theta)(96 - 24\theta^2 \\ & + \theta^4)w_2^1 - 2(4+\theta)(4-\theta)(192 - 64\theta^2 + 3\theta^4)w_1^1 - (2+\theta)(4+\theta)(4-\theta)^2 \\ & (48 - 24\theta - 4\theta^2 + \theta^3)h\big] \big/ \big[(4-\theta^2)((48 + 24\theta + 4\theta^2)^2 - \theta^6)b\big],\end{aligned}$$

and q_2^{1*} and I_2^* 's expressions in (w_1^1, w_2^1) are symmetric.

We then derive suppliers' decisions of the wholesale prices. From supplier 1's perspective, given supplier 2's wholesale price w_2^1, supplier 1 aims to maximize its total profit over two periods as

$$\max_{w_1^1 \geq 0} \left\{ \Pi_{S_1} := \Pi_{S_1}^1(w_1^1, w_2^1) + \Pi_{S_1}^{2*}\big(I_1^*(w_1^1, w_2^1), I_2^*(w_1^1, w_2^1)\big) \right\},$$

where $\Pi_{S_1}^1 = w_1^1\big(q_1^{1*}(w_1^1, w_2^1) + I_1^*(w_1^1, w_2^1)\big)$ and $\Pi_{S_1}^{2*}$ takes the form as in the discussion of period 1 with $I_1 = I_1^*(w_1^1, w_2^1)$ and $I_2 = I_2^*(w_1^1, w_2^2)$. By using the first-order optimality condition, we can obtain the solution to this maximization problem as

$$\begin{aligned}\bar{w}_1^1 = &\big[2(10616832 - 53084160\theta - 86999040\theta^2 + 42393600\theta^3 + 24084480\theta^4 - 11243520\theta^5 \\ &- 271616\theta^6 + 1176320\theta^7 + 142720\theta^8 - 56480\theta^9 - 344\theta^{10} + 124\theta^{11} + 3\theta^{12} - \theta^{13})a \\ &+ 2\theta(4423680 - 3600380\theta^2 + 9789440\theta^4 - 1057280\theta^6 + 52640\theta^8 - 120\theta^{10} + \theta^{12})w_2^1 \\ &- (2+\theta)(4+\theta)(4-\theta^2)(36864 - 368640\theta - 153600\theta^2 + 192000\theta^3 + 6400\theta^4 - 23040\theta^5 \\ &- 16\theta^6 + 88\theta^7 - \theta^9)\big] \,/\, \big[4(10027008 - 83066880\theta^2 + 23214080\theta^4 - 2613760\theta^6 \\ &+ 137600\theta^8 - 336\theta^{10} + 3\theta^{12})\big]\end{aligned}$$

The derivation for $\bar{w}_2^1$ is similar. Then solving the equilibrium results in suppliers' joint decision (w_1^{1*}, w_2^{1*}) as

$$\begin{aligned}w_1^{1*} = w_2^{1*} = &\big[2(10616832 - 53084160\theta - 86999040\theta^2 + 42393600\theta^3 + 24084480\theta^4 \\ &- 11243520\theta^5 - 271616\theta^6 + 1176320\theta^7 + 142720\theta^8 - 56480\theta^9 - 344\theta^{10} + 124\theta^{11} \\ &+ 3\theta^{12} - \theta^{13})a - (2+\theta)(4+\theta)(4-\theta)^2((5013504 + 1474560\theta - 41164800\theta^2 \\ &- 1290240\theta^3 + 11284480\theta^4 + 373760\theta^5 - 1213440\theta^6 - 39040\theta^7 + 59040\theta^8 \\ &+ 160\theta^9 - 128\theta^{10} - 2\theta^{11} + \theta^{12})h\big] \,/\, \big[2(4-\theta)(5013504 + 1474560\theta \\ &- 41164800\theta^2 - 1290240\theta^3 + 11284480\theta^4 + 373760\theta^5 - 1213440\theta^6 - 39040\theta^7 \\ &+ 59040\theta^8 + 160\theta^9 - 128\theta^{10} - 2\theta^{11} + \theta^{12}\big].\end{aligned}$$

Substituting w_1^{1*}, w_2^{1*} back, we can further have

$$\begin{aligned}
I_1^* = I_2^* = \big[& 2(141557760 + 141557776\theta - 1309409928\theta^2 - 123863304\theta^3 + 445317712\theta^4 \\
& + 40550400\theta^5 - 71106656\theta^6 - 6594560\theta^7 + 6010880\theta^8 + 624640\theta^9 - 27776\theta^{10} \\
& - 3424\theta^{11} + 656\theta^{12} + 96\theta^{13} - 6\theta^{14} - \theta^{15})a - (2+\theta)(4+\theta)(4-\theta^2) \\
& (8847360 + 8847360\theta - 70778880\theta^2 - 6635520\theta^3 + 18616320\theta^4 \\
& + 1612800\theta^5 - 1848320\theta^6 - 143360\theta^7 + 80640\theta^8 + 5120\theta^9 - 1520\theta^{10} - 6\theta^{11} \\
& + \theta^{12})\big] \,/\, \big[(2-\theta)(48 + 24\theta - 4\theta^2 - \theta^3)(5013504 + 1474560\theta \\
& - 41164800\theta^2 - 1290240\theta^3 + 11284480\theta^4 + 373760\theta^5 - 1213440\theta^6 - 39040\theta^7 \\
& + 59040\theta^8 + 1600\theta^9 - 1280\theta^{10} - 2\theta^{11} + \theta^{12})\big].
\end{aligned}$$

B.2 One-time Bargaining

For chain i, supplier and retailer bilaterally bargain over the wholesale prices w_i^t and sales quantities q_i^t for both periods $t = 1, 2$ as well as I_i, the amount of inventories carried over between periods, all in one shot at the beginning of period 1, in order to maximize their joint utility established in a generalized Nash bargaining game with retailer's bargaining power vis-a-vis supplier indexed by $\alpha \in [0, 1]$.

Provided that chain 2's wholesale prices w_2^1, w_2^2, sales quantity q_2^1, q_2^2 and s-trategic inventory I_2 are all given, chain 1 will maximize its total utility function as

$$\max_{(w_1^1, w_1^2, q_1^1, I_1) \geq 0,\, q_1^2 \geq I_1} \left\{ (\Pi_{S_1} - D_{S_1})^{1-\alpha} (\Pi_{R_1} - D_{R_1})^{\alpha} \mid \Pi_{S_1} \geq D_{S_1}, \Pi_{R_1} \geq D_{R_1} \right\}, \tag{4.5}$$

where

$$\Pi_{S_1} = w_1^1(q_1^1 + I_1) + w_1^2(q_1^2 - I_1),$$
$$\Pi_{R_1} = p_1^1 q_1^1 + p_1^2 q_1^2 - w_1^1(q_1^1 + I_1) - w_1^2(q_1^2 - I_1) - hI_1,$$
$$D_{S_1} = D_{R_1} = 0.$$

Note that

$$(\Pi_{S_1} - D_{S_1}) + (\Pi_{R_1} - D_{R_1}) = p_1^1 q_1^1 + p_1^2 q_1^2 - hI_1.$$

Using the same argument as in Appendix A.1, we obtain that if $\left(\bar{w}_1^1, \bar{w}_1^2, \bar{q}_1^1, \bar{q}_1^2, \bar{I}_1\right)$ is an optimal solution to (4.5), then we must have

$$\left(\bar{q}_1^1, \bar{q}_1^2, \bar{I}_1\right) = \underset{(q_1^1, I_1)\geq 0,\, q_2^2 \geq I_1}{\arg\max} \left\{p_1^1 q_1^1 + p_1^2 q_1^2 - hI_1\right\}.$$

It is easy to obtain that

$$\bar{q}_1^1 = \frac{(a - \theta b q_2^1)^+}{2b}, \quad \bar{q}_2^2 = \frac{(a - \theta b q_2^2)^+}{2b} \quad \text{and} \quad \bar{I}_1 = 0.$$

A similar argument results in the expression of $\bar{q}_2^1$, $\bar{q}_2^2$ and $\bar{I}_2$ for chain 2, which are symmetric to that for chain 1. Each chain has to make an independent decision on account of the other chain's action. Therefore, solving the corresponding equilibriums leads to the joint decisions of two chains. In equilibrium,

$$\left(q_1^{1*}, q_2^{1*}\right) = \left(q_1^{2*}, q_2^{2*}\right) = \left(\frac{a}{(2+\theta)b}, \frac{a}{(2+\theta)b}\right), \quad (I_1^*, I_2^*) = (0, 0),$$

and correspondingly, the channel profits of two chains are

$$(\Pi_{C_1}, \Pi_{C_2}) = \left(\frac{2a^2}{b(2+\theta)^2}, \frac{2a^2}{b(2+\theta)^2}\right).$$

B.3 Two-time Bargaining

The double-chain two-time bargaining model follows exactly from the single-chain two-time bargaining setting in Section 2.1.2 with the only exception that the market clearing price is modified. The two-period game is modelled as follows.

Period 2: Each chain has to independently make a decision on account of the possible action of the other chain. Given the inventory quantities (I_1, I_2) carried from period 1, the joint decision of two chains is an equilibrium point on the individual best response, which comes from maximizing the total utility function. From chain 1's perspective, if the sales quantity q_1^2 of chain 2 is given, chain 1 aims to maximize the utility function as

$$\max_{w_1^2 \geq 0,\, q_1^2 \geq I_1} \left\{ \left(\Pi_{S_1}^2 - D_{S_1}^2\right)^{1-\alpha} \left(\Pi_{R_1}^{\alpha} - D_{R_1}^2\right)^{\alpha} \mid \Pi_{S_1} \geq D_{S_1}, \Pi_{R_1} \geq D_{R_1} \right\}, \tag{4.6}$$

where

$$\Pi_{S_1}^2 = w_1^2(q_1^2 - I_1), \quad \Pi_{R_1}^2 = p_1^2 q_1^2 - w_1^2(q_1^2 - I_1), \quad D_{S_1}^2 = D_{R_1}^2 = 0.$$

Using the same argument as in Appendix A.1, we obtain that the optimal solution to (4.6) takes the form

$$\bar{w}_1^2 = (1-\alpha)\frac{(a - \theta b q_2^2 - 2bI_1)^+}{2} \quad \text{and} \quad \bar{q}_1^2 = \frac{(a - \theta b q_2^2 - 2bI_1)^+}{2b} + I_1.$$

Similarly, chain 2's response with respect to Chain 1 takes the form

$$\bar{q}_2^2 = \frac{(a - \theta b q_1^2 - 2bI_2)^+}{2b} + I_2.$$

Note that retailers' joint response corresponds to the equilibrium point. We then divide the discussion into four cases.

Case 1:

$$\begin{cases} a - \theta b q_2^2 - 2bI_1 \geq 0, \\ a - \theta b q_1^2 - 2bI_1 \geq 0, \end{cases} \qquad \text{i.e.,} \quad q_1^2, q_2^2 \leq \frac{a - 2bI_1}{\theta b}.$$

By plugging the expression of $\bar{q}_2^2$ into the expression of $\bar{q}_1^2$, we obtain a solution

$$q_1^{2*}(I_1, I_2) = q_2^{2*}(I_1, I_2) = \frac{a}{(2+\theta)b} \quad \text{when} \quad 0 \le I_1, I_2 \le \frac{a}{(2+\theta)b}.$$

For this solution, we further have for $i = 1, 2$,

$$\begin{cases} \Pi_{S_i}^{2*}(I_1, I_2) = (1-\alpha)\dfrac{a^2}{(2+\theta)^2 b} - (1-\alpha)\left(\dfrac{2a}{2+\theta} - bI_i\right) I_i, \\ \Pi_{R_i}^{2*}(I_1, I_2) = \alpha\dfrac{a^2}{(2+\theta)^2 b} + (1-\alpha)\left(\dfrac{2a}{2+\theta} - bI_i\right) I_i. \end{cases}$$

Case 2:

$$\begin{cases} a - \theta b q_2^2 - 2bI_1 < 0, \\ a - \theta b q_1^2 - 2bI_1 \ge 0, \end{cases} \quad \text{i.e.,} \quad q_1^2, q_2^2 \le \frac{a - 2bI_1}{\theta b}.$$

We easily obtain a solution

$$q_1^{2*}(I_1, I_2) = I_1, \quad q_2^{2*}(I_1, I_2) = \frac{a - \theta b I_1}{2b} \quad \text{when} \quad \frac{a}{(2+\theta)b} \le I_1 \le \frac{a}{\theta b},\ 0 \le I_2 \le \frac{a - \theta b I_1}{2b}.$$

For this solution, we further have

$$\begin{cases} \Pi_{S_1}^{2*}(I_1, I_2) = 0, \\ \Pi_{R_1}^{2*}(I_1, I_2) = \left(\dfrac{2-\theta}{2}a - \dfrac{2-\theta^2}{2}bI_1\right) I_1, \\ \Pi_{S_2}^{2*}(I_1, I_2) = (1-\alpha)\dfrac{(a - \theta b I_1)^2}{4b} - (1-\alpha)(a - bI_2 - \theta b I_1)I_2, \\ \Pi_{R_2}^{2*}(I_1, I_2) = \alpha\dfrac{(a - \theta b I_1)^2}{4b} + (1-\alpha)(a - bI_2 - \theta b I_1)I_2. \end{cases}$$

Case 3:

$$\begin{cases} a - \theta b q_2^2 - 2bI_1 \ge 0, \\ a - \theta b q_1^2 - 2bI_1 < 0, \end{cases} \quad \text{i.e.,} \quad q_1^2, q_2^2 \le \frac{a - 2bI_1}{\theta b}.$$

We easily obtain a solution

$$q_1^{2*}(I_1, I_2) = \frac{a - \theta b I_2}{2b}, \quad q_2^{2*}(I_1, I_2) = I_2 \quad \text{when} \quad 0 \le I_1 \le \frac{a - \theta b I_2}{2b},\ \frac{a}{(2+\theta)b} < I_2 \le \frac{a}{\theta b}.$$

For this solution, we further have

$$\begin{cases} \Pi_{S_1}^{2*}(I_1, I_2) = (1-\alpha)\dfrac{(a-\theta b I_2)^2}{4b} - (1-\alpha)(a - bI_1 - \theta b I_2)I_1, \\ \Pi_{R_1}^{2*}(I_1, I_2) = \alpha\dfrac{(a-\theta b I_2)^2}{4b} + (1-\alpha)(a - bI_1 - \theta b I_2)I_1, \\ \Pi_{S_2}^{2*}(I_1, I_2) = 0, \\ \Pi_{R_2}^{2*}(I_1, I_2) = \left(\dfrac{2-\theta}{2}a - \dfrac{2-\theta^2}{2}bI_2\right) I_2. \end{cases}$$

Case 4:

$$\begin{cases} a - \theta b q_2^2 - 2bI_1 < 0, \\ a - \theta b q_1^2 - 2bI_1 < 0, \end{cases} \qquad \text{i.e.,} \quad q_1^2, q_2^2 \leq \frac{a - 2bI_1}{\theta b}.$$

We easily obtain a solution

$$q_1^{2*}(I_1, I_2) = I_1, \quad q_2^{2*}(I_1, I_2) = I_2 \quad \text{when} \quad I_1 > \frac{a - b\theta I_2}{2b}, \ I_2 > \frac{a - b\theta I_1}{2b}.$$

For this solution, we further have

$$\begin{cases} \Pi_{S_1}^{2*}(I_1, I_2) = 0, \\ \Pi_{R_1}^{2*}(I_1, I_2) = (a - bI_1 - \theta b I_2)I_1, \\ \Pi_{S_2}^{2*}(I_1, I_2) = 0, \\ \Pi_{R_2}^{2*}(I_1, I_2) = (a - bI_2 - \theta b I_1)I_2. \end{cases}$$

Period 1: This period is conceptually the same to the double-chain one-time bargaining in Section 3.1.3. But the total utility function in the maximization for deriving each chain's individual best response for establishing the equilibrium is differently. We simply assume that the disagreement point is $(0, 0)$, which means that once the negotiation fails in Period 1, the chain ceases the operation till the end of Period 2. (Indeed, the discussion below remains the same for any constant disagreement point.) Then, we can explicit write out the maximization for chain 1 as

$$\max_{(w_1^1, q_1^1, I_1)\geq 0} \left\{ \left(\Pi_{S_1} - D_{S_1}\right)^{1-\alpha} \left(\Pi_{R_1} - D_{R_1}\right)^{\alpha} \mid \Pi_{S_1} \geq D_{S_1}, \Pi_{R_1} \geq D_{R_1} \right\}, \qquad (4.7)$$

where

$$\Pi_{S_1} := \Pi^1_{S_1} + \Pi^{2*}_{S_1} = w^1_1(q^1_1 + I_1) + \Pi^{2*}_{S_1}(I_1, I_2),$$
$$\Pi_{R_1} := \Pi^1_{R_1} + \Pi^{2*}_{R_1} = p^1_1 q^1_1 - w_1(q^1_1 + I_1) - hI_1 + \Pi^{2*}_{R_1}(I_1, I_2),$$
$$D_{S_1} = 0,\ D_{R_1} = 0,$$

with $\Pi^{2*}_{S_1}(I_1, I_2), \Pi^{2*}_{R_1}(I_1, I_2)$ coming from period 2.

Note that

$$(\Pi_{S_1} - D_{S_1}) + (\Pi_{R_1} - D_{R_1}) = p^1_1 q^1_1 - hI_1 + \Pi^{2*}_{S_1}(I_1, I_2) + \Pi^{2*}_{R_1}(I_1, I_2).$$

Therefore, using the same argument as in Appendix A.1, we obtain that if $\left(\bar{w}^1_1, \bar{q}^1_1, \bar{I}_1\right)$ is a maximizer to (4.7), then we must have

$$\bar{q}^1_1 \in \arg\max_{q^1_1 \geq 0} \left\{\Pi^q_{C_1} := p^1_1 q^1_1\right\}, \quad \bar{I}_1 \in \arg\max_{I_1 \geq 0} \left\{\Pi^I_{C_1} := -hI_1 + \Pi^{2*}_{S_1}(I_1, I_2) + \Pi^{2*}_{R_1}(I_1, I_2)\right\}.$$

It is easy to obtain that

$$\bar{q}^1_1 = \frac{(a - \theta b q^1_2)^+}{2b}.$$

Now we concentrate on the second maximization. Note that $\Pi^I_{C_1}$ is a function of I_1 whose expression depends on the location of I_2. Then we separate the discussion of the maximizer of $\Pi^I_{C_1}$ into two cases with respect to different I_2.

Case I: If $0 \leq I_2 \leq \dfrac{a}{(2+\theta)b}$, then

$$\Pi^I_{C_1} = \begin{cases} \dfrac{a^2}{(2+\theta)^2 b} - hI_1 & \text{if } 0 \leq I_1 \leq \dfrac{a}{(2+\theta)b}, \\ \left(\dfrac{2-\theta}{2}a - \dfrac{2-\theta^2}{2}bI_1 - h\right) I_1 & \text{if } \dfrac{a}{(2+\theta)b} < I_1 \leq \dfrac{a - 2bI_2}{\theta b}, \\ (a - bI_1 - \theta b I_2 - h) I_1 & \text{if } I_1 > \dfrac{a - 2bI_2}{\theta b}. \end{cases}$$

Note that the maximum value of $\Pi^I_{C_1}$ over the third interval is less than the maximum value of $\Pi^I_{C_1}$ over the second interval. Thus, we only need to compare the maximum values of $\Pi^I_{C_1}$ over the first and second intervals. Consider the function $f(t) := \left(\frac{2-\theta}{2}a - \frac{2-\theta^2}{2}bt - h\right)t$. This function f attains its global maximum value $\frac{[(2-\theta)a-2h]^2}{8(2-\theta^2)b}$ at $t = \frac{(2-\theta)a-2h}{2(2-\theta^2)}$. Note that over the first interval $\Pi^I_{C_1}$ attains its maximum value $\frac{a^2}{(2+\theta)^2 b}$ at $I_1 = 0$. Define

$$\bar{h} := \frac{4-\theta^2-\sqrt{8(2-\theta^2)}}{2(2+\theta)}a.$$

Direct calculation yields that $\frac{a^2}{(2+\theta)^2 b} > \frac{[(2-\theta)a-2h]^2}{8(2-\theta^2)b}$ when $h > \bar{h}$. Conversely, when $0 < h < \bar{h}$ (if $\theta > 0$), we have $\frac{a^2}{(2+\theta)^2 b} < \frac{[(2-\theta)a-2h]^2}{8(2-\theta^2)b}$ and also $\frac{a}{(2+\theta)b} < \frac{(2-\theta)a-2h}{2(2-\theta^2)} < \frac{a-2b}{\theta b}$.

Case II: If $I_2 > \dfrac{a}{(2+\theta)b}$, then

$$\Pi^I_{C_1} = \begin{cases} \dfrac{(a-\theta b I_2)^2}{4b} - hI_1 & \text{if } 0 \le I_1 \le \dfrac{a}{(2+\theta)b}, \\ (a - bI_1 - \theta b I_2 - h)I_1 & \text{if } I_1 > \dfrac{a}{(2+\theta)b}. \end{cases}$$

Consider the function $g(t) := (a - bt - \theta b I_2 - h)t$. This function g attains its global maximum value $\frac{(a-\theta b I_2 - h)^2}{4b}$ at $t = \frac{a-\theta b I_2 - h}{2b}$. Thus, the maximum values of $\Pi^1_{C_1}$ over the second interval is less than the maximum values of $\Pi^1_{C_1}$ over the first interval.

Knowing the above, then we can write out the explicit form of the maximizer of $\Pi^I_{C_1}$ as follows.

- If $h > \bar{h}$, then $\bar{I}_1 = 0 \;\forall\, I_2 \ge 0$.
- If $\theta > 0$ and $h = \bar{h}$, then there exists some $I_2^\circ \in \left[0, \frac{a}{(2+\theta)b}\right]$ such that

$$\bar{I}_1 = \begin{cases} 0 \text{ or } \dfrac{(2-\theta)a-2h}{2(2-\theta^2)b} & \forall\, 0 \le I_2 \le I_2^\circ, \\ \dfrac{a-2bI_2}{\theta b} & \forall\, I_2^\circ < I_2 \le \dfrac{a}{(2+\theta)b}, \\ 0 & \forall\, I_2 > \dfrac{a}{(2+\theta)b}. \end{cases}$$

- If $\theta > 0$ and $h < \bar{h}$, then there exists some $I_2^\circ \in \left[0, \frac{a}{(2+\theta)b}\right]$ such that

$$\bar{I}_1 = \begin{cases} \dfrac{(2-\theta)a - 2h}{2(2-\theta^2)b} & \forall\, 0 \le I_2 \le I_2^\circ, \\ \dfrac{a - 2bI_2}{\theta b} & \forall\, I_2^\circ < I_2 \le \dfrac{a}{(2+\theta)b}, \\ 0 & \forall\, I_2 > \dfrac{a}{(2+\theta)b}. \end{cases}$$

We can apply the same argument from chain 2's perspective. Knowing each chain's response to the other, we obtain that in equilibrium,

$$\left(q_1^{1*}, q_2^{1*}\right) = \left(\frac{a}{(2+\theta)b}, \frac{a}{(2+\theta)b}\right),$$

$$(I_1^*, I_2^*) = \begin{cases} (0,0) & \text{if } h > \bar{h}, \\ (0,0), \left(0, \dfrac{(2-\theta)a - 2h}{2(2-\theta^2)b}\right), \left(\dfrac{(2-\theta)a - 2h}{2(2-\theta^2)b}, 0\right) & \text{if } \theta > 0, h = \bar{h} \\ \left(0, \dfrac{(2-\theta)a - 2h}{2(2-\theta^2)b}\right), \left(\dfrac{(2-\theta)a - 2h}{2(2-\theta^2)b}, 0\right) & \text{if } \theta > 0, h < \bar{h}, \end{cases}$$

Correspondingly, the channel profits of Chains 1 and 2 take the form (with the same order as above)

$$\left(\Pi_{C_1}^*, \Pi_{C_2}^*\right) = \begin{cases} (P_1, P_1) & \text{if } h > \bar{h}, \\ (P_1, P_1), (P_2, P_3), (P_3, P_2) & \text{if } \theta > 0, h = \bar{h}, \\ (P_2, P_3), (P_3, P_2) & \text{if } \theta > 0, h < \bar{h}, \end{cases}$$

where

$$P_1 = \frac{2a^2}{(2+\theta)^2 b}, \quad P_2 = \frac{a^2}{(2+\theta)^2 b} + \frac{((4 - 2\theta - \theta^2)a - 2\theta h)^2}{8(2-\theta^2)b},$$
$$P_3 = \frac{a^2}{(2+\theta)^2 b} + \frac{((2-\theta)a - 2h)^2}{8(2-\theta^2)b}.$$

This result indicates that if the storage cost is lower than a certain threshold, strategic inventories do help the channel profits of both chains.

B.4 Leader-follower + Bargaining

With a modification in the price function, the transitive model almost duplicates the setting in Section 2.1.3. The two-period game is then modelled as follows.

Period 2: Follows exactly the discussion of period 2 under cooperation with two-time bargaining in Section 3.1.3 and Appendix B.3.

Period 1: Period 1 is conceptually the same as period 1 under the dynamic leader-follower model in Section 3.1.1, so the discussion is similar to that of period 1 in Appendix B.1. We first derive retailers' joint response, given a presumed pair of wholesale prices (w_1^1, w_2^1) quoted by suppliers. From retailer 1's perspective, if retailer 2's inventory quantity I_2 is given, her goal will be maximizing the total profit over two periods as

$$\max_{(q_1^1, I_1)\geq 0} \left\{ \Pi_{R_1} := \Pi_{R_1}^1 + \Pi_{R_1}^{2*}(I_1, I_2) \right\}.$$

where $\Pi_{R_1}^1 := p_1^1 q_1^1 - w_1^1(q_1^1 + I_1) - hI_1$ and $\Pi_{R_1}^{2*}(I_1, I_2)$ takes the form as at the end of period 2 in Section B.1. This two-dimensional maximization problems can be separated into two one-dimensional maximization problems as

$$\max_{q_1^1\geq 0} \left\{ \Pi_{R_1}^q := (a - bq_1^1 - \theta b q_2^1 - w_1^1)q_1^1 \right\} \quad \text{and} \quad \max_{I_1\geq 0} \left\{ \Pi_{R_1}^I := -(w_1^1 + h)I_1 + \Pi_{R_1}^{2*} \right\}.$$

By the first-order optimality condition, the solution to the first maximization problem is

$$\bar{q}_1^1 = \frac{(a - w_1^1 - \theta b q_2^1)^+}{2b}.$$

A similar argument applied to retailer 2 results in a symmetric expression for the correspondingly quantity $\bar{q}_2^1$. We then solve the equilibrium and obtain the retailers' joint response (q_1^{1*}, q_2^{1*}) as

$$\left(q_1^{1*}(w_1^1, w_2^1), q_2^{1*}(w_1^1, w_2^1)\right) = \left(\frac{(2-\theta)a - 2w_1^1 + \theta w_2^1}{(2+\theta)(2-\theta)b}, \frac{(2-\theta)a - 2w_2^1 + \theta w_1^1}{(2+\theta)(2-\theta)b} \right).$$

Then we focus on the second maximization problem. The discussion becomes much more complicated. Note that $\Pi^I_{R_1}$ is a function of I_1 with its expression dependent on I_2. Then we separate the discussion of the maximizer $\bar{I}_1$ of $\Pi^I_{R_1}$ into two cases with respect to different I_2.

Case i: If $0 \le I_2 \le \dfrac{a}{(2+\theta)b}$, then

$$\Pi^I_{R_1} = \begin{cases} \alpha\dfrac{a^2}{(2+\theta)^2 b} + (1-\alpha)\left(\dfrac{2a}{2+\theta} - bI_1\right) I_1 - (w_1^1 + h)I_1 & \text{if } 0 \le I_1 \le \dfrac{a}{(2+\theta)b}, \\ \left(\dfrac{2-\theta}{2}a - \dfrac{2-\theta^2}{2}bI_1 - w_1^1 - h\right) I_1 & \text{if } \dfrac{a}{(2+\theta)b} < I_1 \le \dfrac{a-2bI_2}{\theta b}, \\ (a - bI_1 - \theta b I_2 - w_1 - h)I_1 & \text{if } I_1 > \dfrac{a-2bI_2}{\theta b}. \end{cases}$$

Note that the values of $\Pi^I_{R_1}$ over the third interval is dominated by the values over the second interval. Thus, we only need to compare the values of $\Pi^I_{R_1}$ over the first and second intervals. Consider the function $f(t) := \left(\frac{2-\theta}{2}a - \frac{2-\theta^2}{2}bt - w_1^1 - h\right)t$. This function f attains its global maximum value $\frac{[(2-\theta)a-2(w_1^1+h)]^2}{8(2-\theta^2)b}$ at $t = \frac{(2-\theta)a-2(w_1^1+h)}{2(2-\theta^2)b}$. Therefore, if $w_1 + h \ge \frac{\theta^2}{2(2+\theta)}a$, then the maximum value of $\Pi^I_{R_1}$ over the second interval is less than the maximum value of $\Pi^I_{R_1}$ over the first interval. Also note that over the first interval $\Pi^I_{R_1}$ attains its maximum value $\frac{a^2}{(2+\theta)^2 b} - \frac{a(w_1^1+h)}{(2+\theta)b} + \frac{(w_1^1+h)^2}{4(1-\alpha)b}$ at $I_1 = \left(\frac{a}{(2+\theta)b} - \frac{w_1^1+h}{2(1-\alpha)b}\right)^+$.

Case ii: If $I_2 > \dfrac{a}{(2+\theta)b}$, then

$$\Pi^I_{R_1} = \begin{cases} \alpha\dfrac{(a-\theta bI_2)^2}{4b} + (1-\alpha)(a - bI_1 - \theta bI_2)I_1 - (w_1^1 + h)I_1 & \text{if } 0 \le I_1 \le \dfrac{a}{(2+\theta)b}, \\ (a - bI_1 - \theta bI_2 - (w_1^1 + h))I_1 & \text{if } I_1 > \dfrac{a}{(2+\theta)b}. \end{cases}$$

Consider $g(t) = (a - bt - \theta bI_2 - (w_1^1 + h))t$. This function g attains its global maximum value $\frac{(a-\theta bI_2-(w_1^1+h))^2}{4b}$ at $t = \frac{a-\theta bI_2-(w_1^1+h)}{2b}$. Note that $\frac{a-\theta bI_2-(w_1^1+h)}{2b} < \frac{a}{(2+\theta)b}$ for any $I_2 \ge \frac{a}{(2+\theta)b}$. Thus, the maximum value of $\Pi^I_{R_1}$

over the second interval is less than the maximum value of $\Pi^1_{R_1}$ over the first interval attained at $I_1 = \left(\frac{a-\theta b I_2}{2b} - \frac{w_1^1+h}{2(1-\alpha)b}\right)^+$.

A similar argument can be applied to retailer 2 to obtain the corresponding inventory quantity $\bar{I}_2$ with the symmetric expression. Therefore, from the above brief discussion, we realize that for certain cases, retailers' joint response could be not unique since multiple equilibrium points (I_1^*, I_2^*) could exist. In such circumstance, suppliers will not be able to decide the optimal pair of wholesale prices due to the unpredictable reaction of retailers. Therefore, when suppliers are making a decision, any candidate pair of wholesale prices leading to unpredictable retailers response will be rejected. Back to the discussion of $\bar{I}_1$ and $\bar{I}_2$, it means that suppliers will only consider the candidate pairs of wholesale prices for which the maximizer of $\Pi^1_{R_1}$ for Case I lying in the first interval, i.e., $\bar{I}_1 \in \left[0, \frac{a}{(2+\theta)a}\right]$, and meanwhile the maximizer of $\Pi^1_{R_2}$ satisfying a symmetric requirement. Under this circumstance, the maximizer $\bar{I}_1$ should have the following expression and the expression of $\bar{I}_2$ should be its symmetry:

$$\bar{I}_1 = \begin{cases} \left(\dfrac{a}{(2+\theta)b} - \dfrac{w_1^1+h}{2(1-\alpha)b}\right)^+ & \text{if } 0 \leq I_2 \leq \dfrac{a}{(2+\theta)b}, \\ \left(\dfrac{a-\theta b I_2}{2b} - \dfrac{w_1^1+h}{2(1-\alpha)b}\right)^+ & \text{if } I_2 > \dfrac{a}{(2+\theta)b}. \end{cases}$$

Correspondingly, retailers' joint response is unique, being of the form

$$I_i^*(w_1^1, w_2^1) = \left(\frac{a}{(2+\theta)b} - \frac{w_i^1+h}{2(1-\alpha)}\right)^+, \quad i = 1, 2.$$

It is easy to find a sufficient condition on the wholesale prices (w_1^1, w_2^1) to guarantee the uniqueness of the above joint response, i.e., $w_i^1 + h \geq \frac{\theta^2}{2(2+\theta)}a$ for $i = 1, 2$. Indeed, the area of (w_1^1, w_2^1) for the uniqueness of $(I_1^*(w_1^1, w_1^2), I_2^*(w_1^1, w_1^2))$ is solvable but complicated. However, the subsequent discussion does not need the explicit form of this area for uniqueness. We only need to verify that the final optimal contract possess the uniqueness at this stage.

We then help suppliers to determine the wholesale prices. From supplier 1's perspective, once supplier 2's wholesale price w_2^1 is given, supplier 1 will maximize its total profit over two periods as

$$\max_{w_1^1 \geq 0} \left\{ \Pi_{S_1} := \Pi_{S_1}^1 + \Pi_{S_1}^{2*} \right\},$$

where $\Pi_{S_1}^1 = w_1^1 \big(q_1^{1*}(w_1^1, w_2^1) + I_1^*(w_1^1, w_2^1) \big)$ and $\Pi_{S_1}^{2*}(I^*(w_1^1, w_2^1))$ takes the form as at the end of period 2 in Section B.1 with $I_1 = I_1^*(w_1^1, w_2^1)$ and $I_2 = I_2^*(w_1^1, w_2^1)$. More specifically,

$$\Pi_{S_1} = w_1^1 \left(\frac{(2-\theta)a - 2w_1^1 + \theta w_2^1}{(2+\theta)(2-\theta)b} + I_1^* \right) + (1-\alpha)\frac{a^2}{(2+\theta)^2 b} - (1-\alpha)\left(\frac{2a}{2+\theta} - bI_1^* \right) I_1^*.$$

Note that Π_{S_1} is a piecewise function with the break point $w_m := \frac{2(1-\alpha)}{2+\theta}a - h$. Let $\Pi_{S_1}^l$ denote the left subfunction Π_{S_1} with $I_1^* = \frac{a}{(2+\theta)b} - \frac{w_1^1+h}{2(1-\alpha)b}$ and let $\Pi_{S_1}^r$ denote the right subfunction with $I_1^* = 0$. Direct calculation yields that $\Pi_{S_1}^l$ attains its maximum at $w_l := \frac{2(1-\alpha)(2(2-\theta)a+\theta w_2^1))}{8(1-\alpha)+(2+\theta)(2-\theta)}$ and $\Pi_{S_1}^r$ attains its maximum at $w_r := \frac{(2-\theta)a+\theta w_2^1}{4}$. Note that the function Π_{S_1} depends on w_2^1. Thus, we discuss the maximizer of Π_{S_1} with respect to different w_2^1. Define

$$A := \left(\frac{8(1-\alpha)}{2+\theta} - (2-\theta) \right) a - \left(4 - \frac{4-\theta^2}{2(1-\alpha)} \right) h, \quad B := \left(\frac{8(1-\alpha)}{2+\theta} - (2-\theta) \right) a - 4h.$$

Note that $A < B$. Then, we have

Case 1: If $w_2^1 \leq A$, then $w_l \leq w_m$, $w_r \leq w_m$, and thus

$$\bar{w}_1^1 = \frac{2(1-\alpha)(2(2-\theta)a + \theta w_2^1))}{8(1-\alpha) + (2+\theta)(2-\theta)}.$$

Case 2: If $A < w_2^1 \leq B$, then $w_l > w_m$, $w_r \leq w_m$, and thus

$$\bar{w}_1^1 = \frac{2(1-\alpha)}{2+\theta}a - h.$$

Case 3: If $w_2^1 > B$, then $w_l > w_m$, $w_r > w_m$, and thus

$$\bar{w}_1^1 = \frac{(2-\theta)a + \theta w_2^1}{4}.$$

Therefore, the quantity $\bar{w}_1^1$, as a function of w_2^1, has 3 pieces if $A > 0$, 2 pieces if $A \le 0 < B$ and only 1 piece if $B \le 0$. The result for $\bar{w}_2^1$ is symmetric.

The disclose of each supplier's individual best response also us the derive the suppliers' joint decision of wholesale prices by straightforward discussion into cases. Define

$$\bar{h}_1 := \frac{2(1-\alpha)\left(2(1-\alpha)(4-\theta)-(4-\theta^2)\right)}{(2+\theta)\left(2(1-\alpha)(4-\theta)+(4-\theta^2)\right)}a \quad \text{and} \quad \bar{h}_2 := \frac{2(1-\alpha)(4-\theta)-(4-\theta^2)}{(2+\theta)(4-\theta)}a.$$

Then we can obtain

- If $0 \le \alpha < \frac{4-2\theta+\theta^2}{2(4-\theta)}$, then

$$w_1^{1*} = w_2^{1*} = \begin{cases} \dfrac{4(1-\alpha)(2-\theta)}{2(1-\alpha)(4-\theta)+(4-\theta^2)}a & \text{if } 0 \le h \le \bar{h}_1, \\ \dfrac{2(1-\alpha)}{2+\theta}a - h & \text{if } \bar{h}_1 < h \le \bar{h}_2, \\ \dfrac{2-\theta}{4-\theta}a & \text{if } h > \bar{h}_2. \end{cases}$$

- If $\alpha \ge \frac{4-2\theta+\theta^2}{2(4-\theta)}$, then

$$w_1^{1*} = w_2^{1*} = \frac{2-\theta}{4-\theta}a \quad \forall\, h \ge 0.$$

Note that for any $\bar{h}_1 < h \le \bar{h}_2$,

$$\frac{4(1-\alpha)(2-\theta)}{2(1-\alpha)(4-\theta)+(4-\theta^2)}a = \frac{2(1-\alpha)}{2+\theta}a - \bar{h}_1 \ge \frac{2(1-\alpha)}{2+\theta}a - h = \frac{2-\theta}{4-\theta}a + \bar{h}_2 - h \ge \frac{2-\theta}{4-\theta}a.$$

Then, one may easily verify that for in any case,

$$w_1^* = w_2^* \ge \frac{2-\theta}{4-\theta}a \ge \frac{\theta^2}{2(2+\theta)}a > \frac{\theta^2}{2(2+\theta)}a - h.$$

Therefore, the sufficient conditions for a unique retailers' equilibrium point (I_1^*, I_2^*) are satisfied. Direct calculation yields the channel profit of both chains as

$$\begin{aligned}
\Pi_{C_i}^* &:= \Pi_{S_i}^* + \Pi_{R_i}^* = p_i^{1*} q_i^{1*} - hI_i^* + \frac{a^2}{(2+\theta)^2 b} \\
&= \frac{2a^2}{(2+\theta)^2 b} - \frac{w_i^{1*} a}{2+\theta} + \frac{(1+\theta)(w_i^{1*})^2}{(2+\theta)^2} - hI_i^* \\
&= \frac{2a^2}{(2+\theta)^2 b} + \frac{1+\theta}{(2+\theta)^2} \left(\frac{2+\theta}{2(1+\theta)} a - w_i^{1*} \right)^2 - \frac{1}{4(1+\theta)} a^2 - hI_i^* \qquad i = 1, 2.
\end{aligned}$$

It is notable that in any case, $\Pi_{C_1}^* = \Pi_{C_2}^* < \frac{2a^2}{(2+\theta)^2 b}$. (The equality can only be achieved if $w_i^{1*} = 0$ or $w_i^{1*} = \frac{2+\theta}{1+\theta} a$ with $I_i^* = 0$, which is impossible.) Moreover,

$$p_i^{1*} = \frac{a - w_i^{1*}}{(2+\theta)b} < \frac{a}{2+\theta)b} \quad \forall\, i = 1, 2.$$

By plugging the detailed expression of (w_1^{1*}, w_2^{1*}), we further have

- If $0 \le \alpha < \frac{4-2\theta+\theta^2}{2(4-\theta)}$, then

$$I_1^* = I_2^* = \begin{cases} \dfrac{\bar{h}_1 - h}{2(1-\alpha)b} & \text{if } 0 \le h \le \bar{h}_1, \\ 0 & \text{if } \bar{h}_1 < h \le \bar{h}_2, \\ 0 & \text{if } h > \bar{h}_2, \end{cases}$$

$$p_1^{1*} = p_2^{1*} = \begin{cases} \dfrac{2(1-\alpha)\theta + (4-\theta)^2}{(2+\theta)(2(1-\alpha)(4-\theta) + (4-\theta^2))b} & \text{if } 0 \le h \le \bar{h}_1, \\ \dfrac{(\alpha+\theta)a + h}{(2+\theta)^2 b} & \text{if } \bar{h}_1 < h \le \bar{h}_2, \\ \dfrac{2a}{(2+\theta)(4-\theta)b} & \text{if } h > \bar{h}_2, \end{cases}$$

$$\Pi_{C_1}^* = \Pi_{C_2}^* = \begin{cases} \dfrac{2a^2}{(2+\theta)^2 b} - \dfrac{(2-\theta)(6+\theta)a^2}{(2+\theta)^2(4-\theta)^2 b} + \Delta(\bar{h}_2 - \bar{h}_1) - \dfrac{h(\bar{h}_1 - h)}{2(1-\alpha)b} & \text{if } 0 \le h \le \bar{h}_1, \\ \dfrac{2a^2}{(2+\theta)^2 b} - \dfrac{(2-\theta)(6+\theta)a^2}{(2+\theta)^2(4-\theta)^2 b} + \Delta(\bar{h}_2 - h) & \text{if } \bar{h}_2 < h \le \bar{h}_2, \\ \dfrac{2a^2}{(2+\theta)^2 b} - \dfrac{(2-\theta)(6+\theta)a^2}{(2+\theta)^2(4-\theta)^2 b} & \text{if } h > \bar{h}_2, \end{cases}$$

where

$$\Delta(t) := \frac{1+\theta}{2+\theta}\left(t^2 - \frac{4+\theta^2}{2(1+\theta)(4-\theta)}t\right).$$

- If $\alpha \geq \frac{4-2\theta+\theta^2}{2(4-\theta)}$, then

$$\begin{aligned}
I_1^* = I_2^* &= \frac{(\bar{h}_2 - h)^+}{2(1-\alpha)b} \quad \forall\, h \geq 0,\\
p_1^{1*} = p_2^{1*} &= \frac{2a}{(2+\theta)(4-\theta)b} \quad \forall\, h \geq 0,\\
\Pi_{C_1}^* = \Pi_{C_2}^* &= \frac{2a^2}{(2+\theta)^2 b} - \frac{(2-\theta)(6+\theta)a^2}{(2+\theta)^2(4-\theta)^2 b} - \frac{h(\bar{h}_2 - h)^+}{2(1-\alpha)b} \quad \forall\, h \geq 0.
\end{aligned}$$

It is interesting to note that for both two cases of α, the chain profit of each chain $\Pi_{C_i}^*$, $i = 1, 2$ increases as the storage cost h increases.

B.5 Bargaining + Leader-follower

With a modification in the price function, the transitive model almost duplicates the setting in Section 2.1.4. The two-period game is then modelled as follows.

Period 2: Follows exactly the discussion of period 2 under the dynamic leader-follower in Section 3.1.1 and Appendix B.1.

Period 1: Period 2 is conceptually the same as period 1 under cooperation with two-time bargaining model in Section 3.1.1, so the discussion is similar to that of period 1 in Appendix B.1 in Section 3.1.3. From chain 1's perspective, if chain 2's sales quantity q_2^1 and inventory quantity I_2 are given, the goal of chain 2 would be maximizing its total utility function, in which the disagreement point should be set to a reasonable outcome once the negotiation fails. Note that if the negotiation fails in Period 1, chain 1 will take no action in period 1 but continue to period 2 under the leader-follower framework on account of the possible strategic inventories of chain 2. Therefore, the disagreement point should be defined as $(D_{S_1}, D_{R_1}) = (\Pi^{2*}_{S_1}(0, I_2), \Pi^{2*}_{R_1}(0, I_2))$, where $\Pi^{2*}_{S_1}$ and $\Pi^{2*}_{R_1}$ take the form as that at the end of Section B.1 (with $I_1 = 0$). Then the maximization problem of chain 1 can be described as

$$\max_{(w_1^1, q_1^1, I_1)\geq 0} \left\{ \left(\Pi_{S_1} - D_{S_1}\right)^{1-\alpha} \left(\Pi_{R_1} - D_{R_1}\right)^{\alpha} \mid \Pi_{S_1} \geq D_{S_1}, \Pi_{R_1} \geq D_{R_1} \right\}, \tag{4.8}$$

where

$$\begin{aligned}
\Pi_{S_1} &:= \Pi^1_{S_1} + \Pi^{2*}_{S_1}(I_1, I_2) = w_1^1(q_1^1 + I_1) + \Pi^{2*}_{S_1}(I_1, I_2),\\
\Pi_{R_1} &:= \Pi^1_{R_1} + \Pi^{2*}_{R_1}(I_1, I_2) = p_1^1 q_1^1 - w_1(q_1^1 + I_1) - hI_1 + \Pi^{2*}_{R_1}(I_1, I_2),\\
D_{S_1} &= \frac{2(4-\theta^2)}{(16-\theta^2)^2 b} \left(\frac{4+\theta}{2+\theta} a - \theta b I_2\right)^2,\\
D_{R_1} &= \frac{4}{(16-\theta^2) b} \left(\frac{4+\theta}{2+\theta} a - \theta b I_2\right)^2.
\end{aligned}$$

with $\Pi^{2*}_{S_1}, \Pi^{2*}_{R_1}$ taking the form as that at the end of Section B.1 (with $I_1 = 0$). Note that

$$(\Pi_{S_1} - D_{S_1}) + (\Pi_{R_1} - D_{R_1}) = \Pi^q_{C_1} + \Pi^I_{C_1},$$

Using the same argument as in Appendix A.1, we obtain that if $\left(\bar{w}_1^1, \bar{q}_1^1, \bar{I}_1\right)$ is an optimal solution to (4.8), then we must have

$$\bar{q}_1^1 \in \arg\max_{q_1^1 \geq 0} \left\{\Pi^q_{C_1}\right\} \quad \text{and} \quad \bar{I}_1 \in \arg\max_{I_1 \geq 0} \left\{\Pi^I_{C_1}\right\}.$$

By the first-order optimality condition, it is not hard to obtain that

$$\begin{aligned}
\bar{q}_1^1 &= \frac{(a - \theta b q_2^1)^+}{2b},\\
\bar{I}_1 &= \frac{(4+\theta)(32 - 8\theta^2 + \theta^4)a - \theta(2+\theta)(32 - 8\theta^2 + \theta^4) b I_2 - (4+\theta^2)(4-\theta)^2 h}{2(2+\theta)(8-\theta^2)(8-3\theta^2) b}.
\end{aligned}$$

A similar argument can also be applied to chain 2. The obtained expression of $\bar{q}_2^1$ and $\bar{I}_2$ is symmetric to that of $\bar{q}_1^1$ and $\bar{I}_1$. Then, solving the equilibrium yields the suppliers joint decision as

$$q_1^{1*} = q_2^{1*} = \frac{a}{(2+\theta)b},$$
$$I_1^* = I_2^* = \frac{(32 - 8\theta^2 + \theta^4)a - (2+\theta)(4+\theta)(4-\theta)^2 h}{(4-\theta^2)(16 + 8\theta - 4\theta^2 - \theta^3)b}.$$

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Abstract

In this thesis we first investigate the existence and the effect of strategic inventories for a single supply chain where the supplier and the retailer bargain for the trading terms. For a two-period problem, we consider both the case of bargaining taking place in both periods and the scenario where the two parties bargain only in one period. We compare our results with those for the scenario where the supplier and the retailer trade under a Stackelberg game framework. We then introduce horizontal competition between supply chains into the system and study how the impact of strategic inventories changes compared to other settings. We have shown that strategic inventories do exist in optimal contracts under most scenarios, and could project different impacts on supply chain performances and profits.

Keywords: Strategic Inventories, Bargaining, Horizontal Competition, Supply Chain Coordination.

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